Note on the determination of
Characteristic Value of Observations

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According to the EN 1504-standards, the EN 206 and the Euronorm ENV 1992-1-1 the lower characteristic value shall be the 5 % fractile and the upper characteristic value shall be the 95 % fractile.

In order to determine the characteristic value of observations obtained from testing concrete or similar materials it is convenient to make the following assumptions:

- The lower characteristic value is defined as the 5 % fractile.
- The upper characteristic value is defined as the 95 % fractile.
- The characteristic value shall be determined from observations at a level of confidence of $\alpha = 84.1$ %.
- The observations from the testing are assumed statistically to be logarithmic normally distributed.
- The coefficient of variation is unknown.

In the case of $n \geq 3$ observations (e.g. strengths) from one single section of inspection, calculation of the characteristic value of the following observations:

$$f_1, f_2, f_3, \ldots, f_n \tag{1}$$

are carried out as follows: first the mean value $M_{\ln f}$ and the standard deviation $S_{\ln f}$ of the Napir logarithm of the observations (1), i.e. the values:

$$\ln f_1, \ln f_2, \ln f_3, \ldots, \ln f_n \tag{2}$$

are carried out. The easiest way is to apply a spreadsheet, e.g. Excel, cf. example 1. Then the lower characteristic value (5 % fractile) is:

$$f_{kl} = \exp\left(M_{\ln f} - k_n \cdot S_{\ln f}\right) \tag{3}$$

and the upper characteristic value (95 % fractile) is:

$$f_{ku} = \exp\left(M_{\ln f} + k_n \cdot S_{\ln f}\right) \tag{4}$$

The factor k_n is based upon the non-central t-distribution and obeys the values shown in Table 1:

n	3	4	5	6	7	8	9	10	11	12	15	20	30	50	100
k_n	4.11	3.28	2.91	2.70	2.57	2.47	2.40	2.34	2.29	2.25	2.16	2.07	1.98	1.89	1.81

Table 1. Values of the factor k_n in equation (3) and (4).

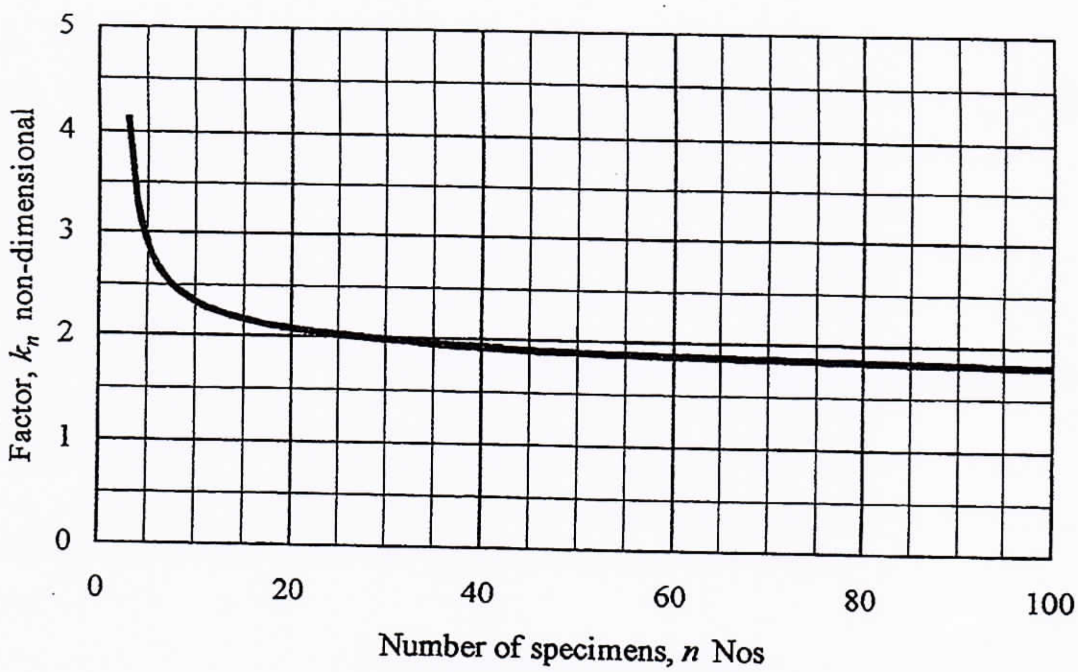

Figure 1. The factor k_n versus the number of observations n.

EXAMPLE 1. The following compressive strengths have been determined by means of the pull-out test method (CAPO-test) from one inspection section:

27.5 25.0 24.5 25.0 22.5 24.0 25.5 28.5 25.0 30.0 MPa

Calculation of the lower characteristic value (5 % fractile) is carried out in the following way, applying a spreadsheet, cf. table 2:

	Compressive strength, f_c MPa	$\ln f_c$
f_{c1}	27.5	3.14186
f_{c2}	25.0	3.21888
f_{c3}	24.5	3.19867
f_{c4}	25.0	3.21889
f_{c5}	22.5	3.11352
f_{c6}	24.0	3.17805
f_{c7}	25.5	3.23868
f_{c8}	28.5	3.34990
f_{c9}	25.0	3.21888
f_{c10}	30.0	3.40120
Mean value	25.75	3.24508
Standard deviation	2.252	0,08576
Coefficient of variation	0.087	—
Lower characteristic value	21.00	—

Table 2. Calculation of the lower characteristic value of observed compressive strength.

In Table 2 the mean value and the standard deviations of the logarithms of the compressive strengths are determined as $M_{\ln f} = 3.24508$ and $S_{\ln f} = 0.08576$ respectively. Thus, the lower characteristic value (5 % fractile) yields:

$$f_k = \exp\left(M_{\ln f} - k_n \times S_{\ln f}\right) = \exp(3.24508 - 2.34 \times 0.08576) = 21.00 \text{ MPa}$$

EXAMPLE 2. In a 450 m^2 overlay casting the following values of pull-off strengths were determined using 75 mm diameter dollies:

1.85 1.91 1.56 1.42 1.88 1.69 MPa

Calculation of the lower characteristic value (5 % fractile) is carried out in the following way, applying a spreadsheet, cf. table 3:

	Pull-off strength, f_t MPa	$\ln f_t$
f_{t1}	1.85	0.6152
f_{t2}	1.91	0.6471
f_{t3}	1.56	0.4447
f_{t4}	1.42	0.3507
f_{t5}	1.88	0.6313
f_{t6}	1.69	0.5247
Mean value	1.718	0.5356
Standard deviation	0.198	0.1187
Coefficient of variation	11.5 %	—
Lower characteristic value	1.240	—

Table 3. Calculation of the characteristic value of observed pull-off strength.

In Table 3 the mean value and the standard deviations of the logarithms of the pull-off strengths are determined as $M_{\ln f} = 0.5356$ and $S_{\ln f} = 0.1187$ respectively. Thus, the lower characteristic value (5 % fractile) yields:

$$f_{tk} = \exp\left(M_{\ln f} - k_n \times S_{\ln f}\right) = \exp(0.5356 - 2.70 \times 0.1187) = 1.240 \text{ MPa}$$

EXAMPLE 3. The chloride measurements by RCT (Rapid Chloride Test) in a 10×25 m bridge slab at a depth of 20-25 mm below the exposed concrete surface gave the following data:

0.160 0.154 0.185 0.176 0.192 0.174 % chloride by mass concrete

Calculation of the upper characteristic value (95 % fractile) is carried out in the following way, applying a spreadsheet, cf. table 4.

In Table 4 the mean value and the standard deviation of the logarithms of the chloride contents are determined as $M_{\ln C} = -1.7545$ and $S_{\ln C} = 0.08405$ respectively. Thus, the upper characteristic value (95 % fractile) yields:

$$C_{uk} = \exp\left(M_{\ln C} + k_n \times S_{\ln C}\right) =$$
$$= \exp(-1.7545 + 2.70 \times 0.08405) = 0.2171 \text{ chloride \% mass concrete}$$

	Chloride, C % mass concrete	ln C
C_1	0.160	– 1.8326
C_2	0.154	– 1.8708
C_3	0.185	– 1.6874
C_4	0.176	– 1.7373
C_5	0.192	– 1.6503
C_6	0.174	– 1.7487
Mean value	0.1735	– 1.7545
Standard deviation	0.0144	+ 0.08405
Coefficient of variation	8.30 %	—
Upper characteristic value	0.2171	—

Table 4. Calculation of the upper characteristic value of observed chloride contents.

EXAMPLE 4. The measurements of the *w/c*-ratio (by thin section technique) of concrete from a pre-casting gave the following data:

0.37 0.38 0.36 (non-dimensional)

Calculation of the upper characteristic value (i.e. 95 % fractile) is carried out in the following way, applying a spreadsheet, cf. table 5.

In Table 5 the mean value and the standard deviation of the logarithms of the w/c-ratios are determined as $M_{\ln w/c} = -1.7545$ and $S_{\ln w/c} = 0.08405$ respectively. Thus, the upper characteristic value (95 % fractile) yields:

$$(w/c)_{uk} = \exp(M_{\ln w/c} + k_n \times S_{\ln w/c}) =$$
$$= \exp(-0.9945 + 4.11 \times 0.02703) = 0.385 \text{ (non-dimensional)}$$

	w/c-ratio, non-dimensional	ln *w/c*
w/c_1	0.37	– 0.9943
w/c_2	0.38	– 0.9676
w/c_3	0.36	– 1.0217
Mean value	0.370	– 0.99450
Standard deviation	0.010	+ 0.02703
Coefficient of variation	2.70 %	—
Upper characteristic value	0.385	—

Table 5. Calculation of the upper characteristic value of observed *w/c*-ratios.