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STATISTICAL METHODS TO EVALUATE IN-PLACE TEST RESULTS

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Abstract
In-place testing is used to estimate the compressive strength of concrete in a structure by measuring another property related to compressive strength. Statistical methods are needed for reliable estimates of in-place strength. Such methods should account for the uncertainties in the measured property, the uncertainty of the correlation relationship, and the variability of the in-place concrete. Standard statistical procedures have not yet been adopted in North American practice. Recommendations are provided for developing the correlation relationship, and a reliable, easy-to-use approach is presented to estimate in-place characteristic strength.

1 Introduction

On April 27, 1978, work was in progress on a hyperbolic reinforced concrete cooling tower located at Willow Island, West Virginia. At about 10:00 a.m., while concrete was being hoisted for the placement of the next 1.5-m lift of the shell, there was a sudden failure of the concrete placed on the previous day. The catastrophic accident resulted in 51 deaths, making it the worst construction accident in U.S. history. A subsequent investigation concluded that the most probable cause of the failure was inadequate concrete strength to support the applied construction loads [1].

This accident demonstrates the critical importance of considering construction loads and monitoring in-place strength development in concrete construction. This incident also provided great impetus in North America to modify construction practices and search for reliable methods for in-place testing of concrete.

In-place tests are used to estimate the strength of concrete in a structure by measuring a property which is related to compressive strength. The compressive strength is usually needed to assess structural capacity using accepted design methods. In-place testing is more economical than testing cores drilled from the structure, and it provides information that is more representative of the concrete in the structure than is obtained from the traditional method of testing field-cured specimens.

In North American practice, in-place testing became a "recognized" alternative to testing field-cured cylinders by the addition of the following sentence to the section of the 1983 ACI Code dealing with form removal [2]:

> "Concrete strength data may be based on tests of field-cured cylinders or, when approved by the Building Official, on other procedures to evaluate concrete strength."

The Commentary to the Code listed acceptable alternative procedures, and further stated that these alternative methods "require sufficient data using job materials to demonstrate correlation of measurements on the structure with compressive strength of molded cylinders or drilled cores." Thus, to use any of the alternative methods, a correlation relationship must be developed to translate the in-place test results to equivalent compressive strength values. In addition, a procedure is needed to analyze in-place test results so that the compressive strength can be estimated with a high degree of confidence. Both steps require statistical analysis of test data.

Procedure for performing in-place tests have been standardized by ASTM [3] and other organizations, and a recent report by the American Concrete Institute [4] provides additional guidance on their use. However, there is no standard practice in North America for the statistical procedures to analyze test results. This paper deals with the key aspects of a suitable statistical procedure. The goal is to propose an initial framework for future efforts to develop a consensus standard for analysis of results. First, the objective of in-place testing during construction is discussed. This is followed by a review of the statistical principles to be considered in applying in-place testing.

2 Background

In designing a reinforced concrete structure, the designer uses the specified compressive strength of the concrete. However, the strength of concrete in a structure is variable and can be described by some type of probability density function. In North American practice, the specified strength is generally taken to represent the strength that is expected to be exceeded with about 90% probability, i.e., if 10 random samples were taken from the structure, 9 of them would be expected to exceed the specified strength. The specified strength is also called the characteristic strength, and this term will be used in subsequent discussion.

The objective of in-place testing during construction is to assure, with a high degree of confidence, that the concrete in the structure is sufficiently strong to resist construction loads. To have a safety margin during construction that is consistent with that under service conditions, one needs to know the in-place characteristic strength at critical stages of construction. How can we be assured of a reliable estimate of the in-place characteristic strength? The answer is by using statistical principles to account for the various sources of uncertainty.

The application of in-place testing for monitoring strength development during construction involves the following steps:

- Establish the correlation relationship between the in-place test result and compressive strength.
- Perform the in-place tests during critical stages of construction.
- Analyze the test results to obtain a reliable estimate of the in-place characteristic strength.

The statistical principles to be considered during these steps are discussed.

3 Correlation relationship

Some manufacturers of in-place testing equipment provide correlation relationships along with their devices. However, the ASTM standards [3] and the ACI report [4] recommend that a correlation relationship should be established for the specific test instrument using the concrete materials that will be used in construction. The following questions must be answered to develop a reliable correlation relationship:

- How many test points (i.e., strength levels) are needed?
- How many replicate tests should be performed at each strength level?
- How should the data be analyzed?

3.1 Number of test points

The usual procedure is to perform replicate in-place tests and replicate standard compression tests[a] at various strength levels. The average values of the results are used to establish the correlation relationship using regression analysis. Because the correlation relationship will be used subsequently to estimate compressive strength from in-place test results, compressive strength is treated as the dependent variable (Y-value) and the in-place result as the independent variable (X-value).

The first principle to consider in planning the correlation testing program is that the tests should have coverage. This means that the range of strengths should cover the range of strengths anticipated in the structure during the period when strength will be estimated. The correlation relationship must never be used for extrapolation. Therefore, if very low in-place strengths are to be estimated, such as might be required during slipforming, the testing program must include these low strength levels. The second principle to consider is that the test points should have balance, which means that they should be evenly spaced.

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[a] Compressive strength is considered in this discussion, but correlation could also be established between in-place test results and other standard strength properties, such as flexural strength or indirect tensile strength.

The number of strength levels that should be used depends on econom[ic] considerations and on the acceptable level of precision. To have an understanding of the effect of the number of test points on the precis[ion] of the correlation relationship, it is useful to examine the residual standard deviation (standard error) of the best-fit correlation relationship. The residual standard deviation is the basic statistic used [to] quantify the uncertainty of the calculated relationship. For a linear correlation relationship, an unbiased estimate of the residual standar[d] deviation is as follows:

$$S_e = \sqrt{\frac{\sum (d_{yx})^2}{n - 2}} \quad (1$$

where,

S_e = residual standard deviation,
d_{yx} = deviation of each test point from the best-fit line, and
n = number of test points.

For the sake of discussion, assume that each test point deviates from th[e] best-fit line by the same amount, δ, and that this deviation is independent of the number of points. The estimated residual standard deviation would equal:

$$S_e = \delta \sqrt{\frac{n}{n - 2}} \quad (2)$$

When the correlation relationship is used to estimate the mean value of Y at X, the width of the confidence interval for the estimate is related to the residual standard deviation by the following expression [5]:

$$W = 2\, t_{n-2,\alpha/2}\, S_e \sqrt{\frac{1}{n} + \frac{(X - X_a)^2}{S_{xx}}} \quad (3)$$

where,

W = the $(1-\alpha)$ confidence interval for the estimated mean value of Y for the value X,
$t_{n-2,\alpha/2}$ = Student t-value for n-2 degrees of freedom and significance level = $\alpha/2$,
X_a = grand average of X-values used to develop correlation relationship,
S_{xx} = sum of squares of deviations of X-values from X_a.

The second term under the square root sign in Eq. (3) shows that the confidence interval increases as the value of X is farther from the grand average X_a.

To examine qualitatively how the width of the confidence interval is affected by the number of test points, consider the case where $X = X_a$, so that the second term under the square root sign in Eq. (3) equals zero. Substituting Eq. (2) into Eq. (3), we obtain the following expression for the confidence interval:

$$W = 2\, t_{n-2,\alpha/2}\, \delta \sqrt{\frac{1}{n}} \ldots$$

Figure 1(a) shows the variation W as a function of the number of points for a 95% confidence level. It is seen that, for a small number of test points, by including an additional test point there is a significant reduction in the confidence interval. However, for a large number of points, the reduction is small. Therefore, the appropriate number of strength levels is determined by considerations of economy and precision. Figure 1(b) shows the reduction in the confidence interval by using n test points compared with using (n-1) points. From this figure it is reasonable to conclude that the minimum number of test points is six, while more than nine tests would probably not be economic.

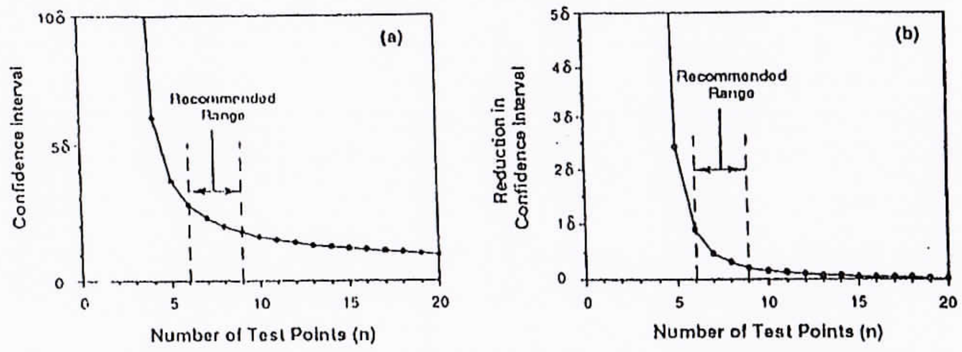

Fig. 1 (a) Effect of number of test points on the 95% confidence interval at $X = X_a$; (b) decrease in confidence interval for n test points compared with (n-1) test points.

In summary, in planning the correlation testing program, six to nine strength levels should be used. The strength levels should be evenly spaced, and the strength range should include the anticipated strengths to be estimated in the field.

3.2 Number of replications

The number of replicate tests at each strength level affects the uncertainty of the average values. The standard deviation of the average value varies with the inverse of the square root of the number of replicate tests used to obtain the average. The effect of the number of tests in the precision of the average is similar to that shown in Fig. 1(a). Thus the number of replicate tests is also governed by considerations of precision and economy.

The required number of replicate tests depends on the within-test variability of the method (also called repeatability), the allowable error between the sample average and the true average, and the confidence level that the allowable error is not exceeded. This number can be established from statistical principles (e.g., ASTM E 122, Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process). Alternatively, the number can be based upon accepted practice. For acceptance testing, ACI 318 [2] considers a test result as the average strength of two molded cylinders. Therefore, for the correlation testing, two replicate standard compression tests are adequate for measuring the average strength at each level.

The number of companion in-place tests at each strength level is governed by the competing factors of precision and economy. The ACI report [4] suggests using a number of replicates such that the average values of the in-place test results have a comparable precision to the average compressive strength. If the coefficients of variation of the standard compression test and of the in-place test method are CV_y and CV_x, respectively, the ratio of the number of tests should be:

$$\frac{n_x}{n_y} = \left(\frac{CV_x}{CV_y} \right)^2 \qquad (5)$$

where,

n_x = number of replicate in-place tests, and
n_y = number of replicate compression tests.

Table 1 lists recommended values of within-test coefficients of variation for various tests [4]. These values may be used for planning a correlation testing program. For example, for correlation testing involving cylinders and the pullout test, the replicate number of pullout tests at each strength level would be $2 \cdot (8/4)^2 = 8$.

3.3 Test specimens

Ideally, it would be desirable to determine the compressive strength and the in-place result on the same specimen. Unfortunately, this is only possible with those methods that are truly nondestructive, such as pulse velocity and rebound number. For other methods, separate specimens are needed for measuring compressive strength and the in-place test result. The ACI report provides recommendations on the type of specimens for pullout and probe penetration tests [4].

Table 1 Within-test variability for various test methods [4]

Test Method	Within-Test Coefficient of Variation (percent)
Cylinder Compression (ASTM C 39)	4
Core Compression (ASTM C 42)	5
Pullout (ASTM C 900)	8
Probe penetration (ASTM C 803)	5
Rebound hammer (ASTM C 805)	12
Pulse velocity (ASTM C 597)	2

It is important that companion specimens are tested at the same maturity. This is especially critical for early-age tests, when strengths vary significantly with age and are dependent on the previous thermal history. The problem arises because of differences in early-age temperature rise in specimens of different geometries. A simple approach for moderating differences in specimen temperatures is to cure all specimens in water baths. The high heat capacity of ...

heat of hydration without a significant rise in tempe[rat]ure. Failure to perform compression and in-pace tests on companion specimens of equal maturity will result in an inaccurate correlation relationship, which will lead to systematic errors (or biases) when used to estimate the in-place strength in a structure.

3.4 Regression analysis

After the data are obtained, the correlation relationship is determined by regression analysis of the average test results at each strength level. Historically, most correlation relationships have been modeled as straight lines, and ordinary least squares (OLS) analysis has been used to estimate the slopes and intercepts of the lines. However, OLS-analysis is based on two assumptions:

- There is no error in the X-value.
- The error (standard deviation) in the Y-value is constant.

The first of these assumptions is violated because, as shown in Table 1, in-place tests (X-value) generally have greater within-test variability than compression tests (Y-value). In addition, it is generally accepted that the within-test variability of standard cylinder compression tests is described by a constant coefficient of variation [7]. Therefore, the standard deviation increases with increasing compressive strength, and the second assumption is also violated. The end result is that OLS-analysis leads to errors in the estimated parameters of the correlation relationship. There are approaches for dealing with these problems.

The problem of increasing variability of compressive strength as the average strength increases is discussed first. If test results having a constant coefficient of variation are transformed by taking their natural logarithms, the standard deviation of the logarithmic values will be constant [7]. Thus the second assumption of OLS can be satisfied by regression analysis using the average of the logarithms of the test results at each strength level. If a linear relationship is used, it would be as follows:

$$\ell n\ C = a + \beta\ \ell n\ I \qquad (6)$$

where,

$\ell n\ C$ = average of natural logarithms of compressive strengths,
a = intercept of line,
β = slope of line, and
$\ell n\ I$ = average of natural logarithms of in-place test results.

By taking the anti-logarithm, Eq. (6) becomes a power function:

$$= e^{a}\ I^{\beta} = A\ I^{\beta} \qquad (7)$$

The exponent, β, determines the degree of non-linearity of the untransformed correlation relationship. If β=1, the correlation relationship is a straight line passing through the origin with a slope = A. If $\beta \neq 1$, the relationship is curved upward or downward, depending on whether β is greater than or less than one. Regression analysis using the natural logarithms of the test results provides two benefits: (1) it satisfies one of the underlying assumptions of OLS-analysis, and (2) it allows a non-linear correlation relationship, if it is needed.

Regression analysis which accounts for X-error can be performed w[ith] little additional effort compared with OLS-analysis. One such proce[dure] was proposed by Mandel [8] and was used by Stone and Reeve [9] to dev[elop] a rigorous procedure for estimating the in-place characteristic stre[ngth]. Mandel's approach involves the use of a parameter, λ, which is define[d] as the variance (square of the standard deviation) of the Y-variable divided by the variance of the X-variable. For the correlation testi[ng] program, the value of λ would be obtained from the squares of the ave[rage] (pooled)[b] standard deviations of the average compressive strengths an[d] in-place results. If the number of replications have been chosen so that average values are measured with comparable precision, the value λ should be close to one.

Mandel's procedure also allows for the possibility of correlation between the errors associated with the average X- and Y-values. For [the] case where the compression tests and in-place tests are performed on separate specimens, there should be no correlation between the errors. For the case where tests are performed on the same specimens (e.g., using the rebound method), there could be some correlation. In this study, it was assumed that there was no correlation between the errors.

Mandel's procedure is explained in Ref. [8] and will not be repeate[d] here in detail; however, the principle is discussed. The parameter λ and the correlation test results (averages of the logarithms of the in-place results (X-values) and average of logarithms of compressive stre[n]gths (Y-values)) are used to calculate two constants k and b. These calculations use the usual sum of squares and cross-products used in OLS-analysis. The constants k and b are used to transform the X- and [Y-] values into U- and V- values as follows:

$$U = X + k\ Y \qquad (8a)$$
$$V = Y - b\ X \qquad (8b)$$

The key feature is that this transformation results in U- and V-values which satisfy the two assumptions for OLS-analysis, and a straight line is fitted to the transformed data using OLS-analysis. The results are transformed back to the (X,Y) scales, and one obtains estimates of a an[d] β in Eq. (6) along with the standard deviations of the estimates. The estimate of β is equal to the constant b determined in the initial step of the analysis. The most important feature of Mandel's analysis is that the estimated standard deviation of the predicted value of Y for a new value of X accounts for error in the new X-value as well as the error of fit. The procedure can be implemented using a hand-held programmable calculator or a spreadsheet program as was done for this paper.

The difference between OLS-analysis and Mandel's procedure is illustrated in Fig. 2. In OLS-analysis, the best-fit straight line is the

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[b] If the same number of replicate tests are used at each strength level, the average standard deviation is used. If the numbers differ because of rejection of outliers, the pooled standard deviation must be calculated accounting for the number of replicates at each strength level [8].

one which minimizes the sum of squares of the vertical deviations of the data points from the line, as shown in Fig. 2(a). On the other hand, Mandel's analysis minimizes the sum of squares of deviations along a direction inclined to the straight line, as shown in Fig. 2(b). The inclination of the direction of minimization depends on the value of λ. As λ decreases, there is more error in the X-values, and the angle θ in Fig. 2(b) increases.

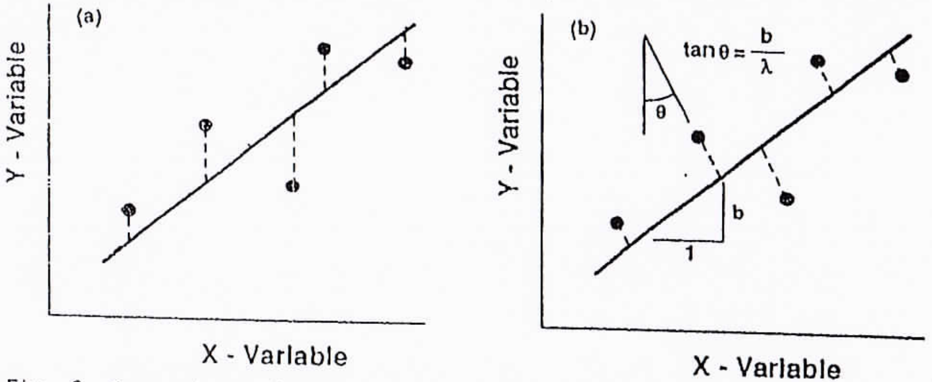


Fig. 2 Comparison of least-squares fit using (a) OLS-analysis and (b) Mandel's method [8].

Table 2 Average of natural logarithms of pullout-loads and cylinder compressive strengths; concrete made with 19-mm river gravel [10]

Pullout Load(P) (kN)	Compressive Strength (C) (MPa)	ℓn (P)	ℓn(C)
9.72	10.41	2.274	2.343
12.18	14.21	2.500	2.654
16.62	18.69	2.811	2.928
19.97	22.83	2.994	3.128
27.13	28.34	3.301	3.344
30.12	31.66	3.405	3.455
34.09	40.07	3.529	3.691
36.00	42.90	3.584	3.759
Pooled S.D.		0.112	0.037

Results from a pullout-test correlation study [10] performed at the U.S National Bureau of Standards (NBS) will be used to illustrate the difference between the results of OLS-analysis and Mandel's procedure. Table 2 lists the averages of the natural logarithms of 11 pullout tests and five cylinder compression tests performed at each of eight strength levels. Also shown are the pooled standard deviations of the natur logarithms of the replicate test results. The value of λ is as fol

$$\lambda = \frac{\frac{(0.037)^2}{5}}{\frac{(0.112)^2}{11}} = 0.24$$

In this case, λ is less than one because five replicate compression were performed at each strength level, which resulted in less error the average compressive strength compared with the average pullout l

The logarithm values in Table 2 were used to obtain the best-fit straight lines using OLS-analysis and Mandel's procedure with λ= 0.24 The results are shown in Table 3. The data and best-fit lines are pl ted in Fig. 3(a). The lines obtained by the two regression methods a practically identical and cannot be distinguished in Fig. 3(a). So i might be argued that there is no need to resort to the more rigorous method of analysis since the same best-fit line is obtained. However this argument does not consider the uncertainty associated with the estimate of Y for a new X. For example, consider the case where the average of the natural logarithm of 11 pullout tests is 3.0 and the standard deviation is 0.11. Table 3 shows that the estimated values the logarithms of compressive strength are equal, but Mandel's proced results in greater uncertainty in the estimate. The standard deviatic of the estimate and the appropriate Student's t-value can be used to calculate the lower confidence limit of the average compressive streng Table 3 shows the lower limits for a 5% significance level. Thus ac- counting for the X-error results in a lower value for the estimated Y a particular confidence level.

Table 3 Results of regression analysis of data in Table 2

	OLS	Mandel
a	0.065	0.037
S_a	0.134	0.134
β	1.016	1.025
S_β	0.043	0.044
Y at X=3.0	3.112	3.112
SD_Y at X=3.0	0.020	0.054
5% lower limit	3.074 (21.6 MPa)	3.006 (20.2 MPa)

Figure 3(a) compares the lower confidence limits obtained with the tw analysis procedures. In Fig. 3(b), the results have been converted back into real units. It is seen that the uncertainty of the estimated aver- age compressive strength increases with increasing concrete strength. This is because the within-test coefficient of variation is assumed to be constant.

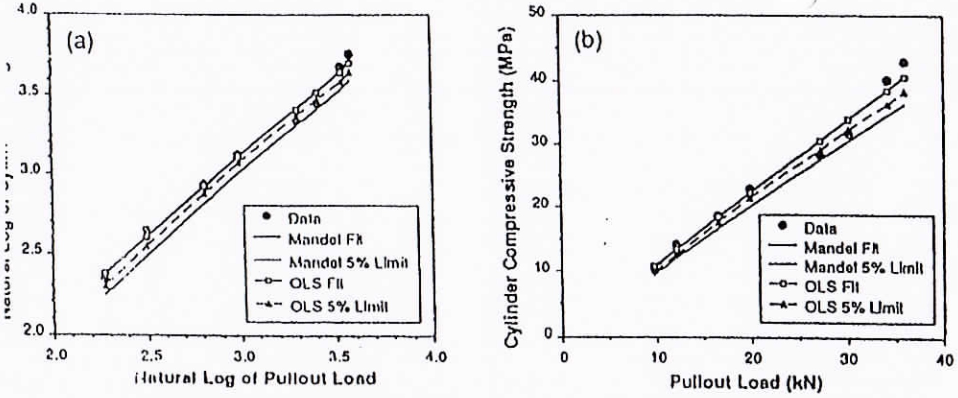


Fig. 3 Correlation relationship and lower confidence limits (0.05 significance level) using OLS-analysis and Mandel's analysis: (a) natural logarithm transformation and (b) real units.

In summary, it is recommended that regression analysis should be performed using the natural logarithms of the test results. This will accommodate the increase in within-test variability with increasing compressive strength. By so doing, the correlation data is fitted with a power function rather than a straight line. Use of the transformed data implies that concrete strength is distributed as a log-normal rather than a normal distribution. It has been argued that possible errors from this assumption are not significant [9]. Regression analysis should be performed using a procedure that accounts for X-error. Failure to account for X-error results in a low estimate of the uncertainty associated with estimates of average compressive strength.

4 Estimate of in-place characteristic strength

Having established the correlation relationship and its associated uncertainty, the next step is to use this relationship and the results of tests performed on the structure to estimate the in-place strength of the concrete. The objective is to estimate the characteristic strength which has a high probability of being exceeded. Thus it is necessary to account for the uncertainties of the correlation relationship and of the average value of the in-place test results.

The first question that has to be answered is how many in-place tests are required to estimate the characteristic strength in a given placement. As was the case in selecting the number of replicate tests for correlation testing, the number of in-place tests is influenced by the desired precision and by economy. However, there are now two sources of variability to consider: (1) the within-test variability of the test method and (2) the batch-to-batch variability of the concrete in the structure. The ideal case would be to perform enough tests at a single location so that the average in-place result is measured with sufficient precision at that location, and there should be sufficient test locations so that the average property of the concrete in the structure is measured with sufficient precision. For a test like the rebound number method, this approach could be used. However, for methods such as pullout, probe penetration, or break-off, such a scheme would result in a prohibitively expensive testing program.

To arrive at a practical answer to the question of how many tests, we can rely on the accepted practices for quality control based on standard-cured specimens. Codes of practice generally require a minimum number of standard tests per specified volume of concrete in the placement. Thus the number of in-place tests in a given placement can be determined by using Eq. (5) and Table 1. This concept has been discussed further in Ref. [10].

After the in-place tests are performed, the final step is to analyze the results and estimate the characteristic strength in the structure. This is probably the most critical phase of an in-place testing program, but it has not been studied as much as other aspects of in-place testing. One of the procedures that is being used is based on the concept of the tolerance limit [11]. The lower tolerance limit represents the value that is expected to be exceeded by a certain proportion of the population with a prescribed confidence level [5]. Thus the characteristic strength, i.e., the strength expected to be exceeded by 90% of concrete in the structure, would be:

$$C_{0.1} = C_a - K_{0.9,\gamma} \; S_{cf} \qquad (9)$$

where,

$C_{0.1}$ = characteristic strength,
C_a = average compressive strength,
$K_{0.9,\gamma}$ = one-sided tolerance factor for proportion 0.9 and confidence level γ, and
S_{cf} = standard deviation of concrete strength in the structure.

In applying the tolerance limit approach to evaluate in-place test results [11], the average concrete strength is obtained by using the average of the in-place results and the correlation relationship. The value of S_{cf} is taken as the standard deviation of the individual strengths estimated for each in-place test result. The tolerance limit approach has been criticized [10] because: Eq. (9) does not account for the uncertainty in the correlation relationship, and the approach assumes that the variability of concrete strength equals the variability of the in-place test results. Thus it felt that Eq. (9) does not have a sound statistical basis for evaluating in-place test results.

To overcome the deficiencies of the tolerance limit approach, the NBS developed a rigorous statistical procedure to estimate the in-place characteristic strength [9]. Basically, the NBS procedure estimates the expected characteristic strength and its uncertainty. From these estimates, one determines the value of the characteristic strength that is expected to be exceeded with a desired level of confidence. The procedure is complex, and it is well-suited for implementation on a personal computer. The characteristic strengths computed by using the rigorous approach were compared with the values computed by using the tolerance limit method [10]. The comparison showed that the tolerance limit approach resulted in lower estimates of in-place characteristic strength, especially when the variability of the in-place results was high. While this may be acceptable for safety, it can lead to unnecessary delays in the construction schedule.

A simpler technique than the rigorous NBS method was developed which accounts for the various sources of uncertainty in estimating the in-place characteristic strength [12]. The simplified method was designed for use with spreadsheet software and was shown to be slightly conservative compared with the rigorous method. Although it was shown to be adequate, the simplified method was not rigorous because OLS-analysis was used, and an empirical approach was used to arrive at the procedure to compute the value of the characteristic strength for a 95% confidence level.

In this paper, a new method is proposed which retains the main features of the rigorous NBS method, but which can be readily implemented using spreadsheet software. The basic approach in the new method is illustrated in Fig. 4(a). Mandel's procedure is used to obtain the correlation relationship and its uncertainty. The results of the in-place tests are used to compute the lower 5% confidence limit for the estimated average in-place strength. The characteristic strength is determined assuming a log-normal distribution for the in-place concrete strength. Calculations are performed using natural-logarithm values, and the last step is to convert the estimated characteristic strength into real units.

The lower 5% confidence limit is obtained using Mandel's formula [8] for the standard deviation of an estimated value of Y for a new X. The formula has been modified according to Ref. [9] so that it incorporates the variability of the average in-place result from tests on the structure. The standard deviation is:

$$S_Y^2 = \left[\frac{1}{n} + (1 + k\,b)^2 \frac{(X - X_a)^2}{S_{uu}} \right] S_e^2 + b^2 S_X^2 \tag{10}$$

where,

S_Y = standard deviation of estimated value of Y (average concrete strength),
n = number of points used to obtain the correlation relationship,
b = estimated slope of the correlation relationship,
k = b/λ, where λ is obtained from the within-test variability during correlation testing,
X = average of results of in-place tests performed on the structure,
X_a = average of X-values during correlation tests,
S_e = residual standard deviation of correlation relationship,
S_{uu} = sum of the squares of the deviations of the U-values (Eq. 8(a)) from the average, and
S_X = standard deviation of average of in-place tests performed on the structure.

It is seen that the S_Y is composed of two parts: the first part considers the uncertainty of the correlation relationship and the second part considers the variability of the in-place test results obtained from testing the structure. Because Eq. (10) is the sum of two variances which may have different degrees of freedom, a formula has been suggested for computing the effective degrees of freedom for S_Y [9]. For simplicity, this new method assumes that there are (m-1) degrees of freedom for S_Y, where m is the number of in-place tests performed on the structure.

The lower confidence limit for the average concrete strength is as follows:

$$Y_{low} = Y - t_{\nu,\alpha}\, S_Y \tag{11}$$

where,

Y_{low} = lower confidence limit at significance level α,
Y = transformed average concrete strength for new value of X (the average of the in-place results),
$t_{\nu,\alpha}$ = Student's t-value for ν = m-1 degrees of freedom and significance level α.

The next step is to estimate the transformed characteristic strength based on the average strength obtained from Eq. (11). It is assumed tha the in-place compressive strength follows a log-normal distribution, so that the characteristic strength is computed as follows:

$$Y_{0.9} = Y_{low} - 1.282\, S_{cf} \tag{12}$$

where,

$Y_{0.9}$ = logarithm of strength expected to be exceeded by 90% of the population,
S_{cf} = standard deviation of logarithm of concrete strength in the structure.

The value of S_{cf} is obtained from the assumption that the ratio of the standard deviation of compression strength to the standard deviation of in-place test results has the same value in the field as was obtained during the laboratory correlation testing [8,9,12]. Thus the following relationship is assumed:

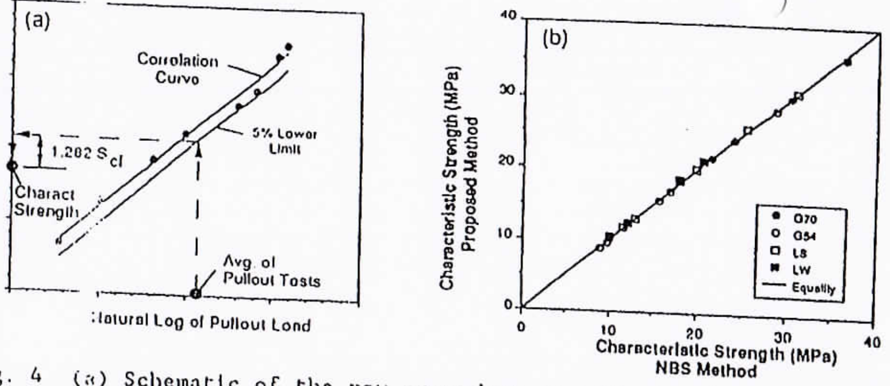
$$\frac{S_{cf}}{S_{if}} = \frac{S_{cl}}{S_{il}} \tag{13}$$

where,

S_{cf}, S_{cl} = standard deviation of logarithms of compressive strength in the field and laboratory, respectively, and
S_{if}, S_{il} = standard deviation of logarithms of the in-place results in the field and in the laboratory, respectively.

The final step is to convert the result obtained from Eq. (12) into real units by taking the anti-logarithm.

To evaluate the new method compared with the rigorous NBS procedure, a simulation was performed using the pullout test. Ten replicate pullout loads were randomly generated for three strength levels and for two levels of variability [10]. These simulated test results were used to obtain the characteristic strength (5% significance level) by the procedure described above (see also Fig. 4(a)). The calculations were performed for the four sets of correlation data reported in Ref. [10], and the results are summarized in Table 4. Figure 4(b) compares the characteristic strengths estimated by the two procedures. It is seen that the proposed method gives nearly the ...



ig. 4 (a) Schematic of the new procedure to estimate characteristic trength; (b) comparison of estimates using NBS method and new procedure.

able 4 Comparison of in-place characteristic strengths using proposed method and NBS method [9]

Test eries† [1]	In-Place Pullout Results Avg. Load (kN) [2]	C.V. (%) [3]	Pred. Avg. Str. (MPa) [4]	Characteristic Strength NBS Method 5% Risk (MPa) [5]	Characteristic Strength Proposed Method (MPa) [6]	Difference (6-5)/5 (%) [7]
70	13.39	10.9	14.8	13.0	13.0	0.0
70	13.48	20.7	14.7	11.7	11.7	0.0
70	22.12	11.9	24.6	21.6	21.5	-0.3
70	22.52	20.3	24.8	19.9	19.9	0.0
70	36.13	9.9	40.8	36.3	36.0	-0.8
70	36.09	21.9	39.8	30.3	30.3	0.0
54	13.39	10.9	11.6	9.8	9.7	-1.4
54	13.48	20.7	11.5	9.0	8.9	-0.8
54	22.12	11.9	19.6	16.9	16.8	-0.8
54	22.52	20.3	19.7	15.7	15.6	-0.4
54	36.13	9.9	32.9	28.7	28.6	-0.5
54	36.09	21.9	32.1	24.1	24.1	0.0
LS	13.39	10.9	14.9	13.0	13.0	0.0
LS	13.48	20.7	14.9	11.6	11.7	1.2
LS	22.12	11.9	23.0	19.9	20.0	0.7
LS	22.52	20.3	23.1	18.1	18.3	1.5
LS	36.13	9.9	35.3	31.1	31.2	0.4
LS	36.09	21.9	34.6	25.4	25.9	1.9
LW	13.39	10.9	15.0	12.1	12.3	1.1
LW	13.48	20.7	14.9	10.1	10.5	3.4
LW	22.12	11.9	26.1	20.6	21.0	1.7
LW	22.52	20.3	26.2	17.9	18.4	3.1

†G70 = river gravel aggregate, apex angle = 70°; G54 = river gravel aggregate, apex angle = 54°. LS = limestone aggregate and LW = lightweight aggregate, both with apex angle = 70°.

5 Summary

This paper has discussed statistical methods to estimate the characteristic strength of concrete based on the results of in-place tests. Mandel's regression analysis procedure which accounts for error in the X-value has been reviewed. The procedure should be used to obtain the correlation relationship for the particular construction project. Regression analysis should be performed using the natural logarithms of the results obtained from the correlation testing program; this accounts for the increasing within-test variability with increasing concrete strength. While ordinary-least-squares analysis results in approximately the same correlation relationship as with Mandel's procedure, the standard deviation of the predicted value of Y for a new value of X is underestimated using OLS-analysis.

A simplified version of the rigorous NBS method to estimate the in-place characteristic strength is proposed. Based on simulated in-place test data, the simplified approach yields estimates which are very close to those obtained by the NBS method. The calculations were performed by modifying a spreadsheet template previously developed for OLS-analysis [12]. The proposed method includes the key aspects of the rigorous procedure but is easier to implement. Thus it is felt that a balance has been attained between statistical rigor and practicality.

6 References

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