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Statistical Methods for In-Place Strength Predictions by the Pullout Test

by William C. Stone, Nicholas J. Carino, and Charles P. Reeve

out tests and companion cylinder tests were conducted to exami ariations in ultimate load with respect to concrete strength, and to provide experimental data for the development of a new statistical procedure for predicting in-place compressive strength from the pullout test. The coefficients of variation were found to be relatively constant with average values of 4 percent for cylinder tests and approximately 10 percent for pullout tests in concrete with hard coarse aggregates. Pullout tests in lightweight concrete exhibited coefficients of variation of only 6 percent. Two test series were conducted with river gravel aggregate using apex angles of 54 and 70 deg. Both geometries produced coefficients of variation of 10 percent. A statistical procedure is developed for determining the correlation equation which accounts for: 1) the constant coefficients of variation in ultimate load, and 2) the X-variable (pullout load) error. A procedure is also developed to predict the in-place characteristic strength to any desired confidence level. A method is presented to determine the appropriate number of in-place tests to be performed for a given concrete placement. A recommended minimum number of 8 to 12 pullout tests per 76 cubic meters (100 cubic yards) is proposed.

ords: aggregates; compressive strength; concrete construction; lightweight ggates; pullout tests; regression analysis; statistical analysis.

This paper is the fourth in a series detailing a study performed at the National Bureau of Standards (NBS) on the pullout method of in-place strength evaluation of concrete. The principal objective of the initial research[1,2] was to obtain a fundamental understanding of the mechanics of failure in an effort to resolve the question of what strength property of concrete is measured by the pullout test.

The objective of the second phase of the NBS program[3] was to determine the effect of changes in the geometry of the test apparatus and the effect of various concrete aggregate properties on the reliability of the test. This test series gave rise to two important questions: For a given test geometry and mix, was the variability of the pullout strength uniform (i.e., a constant standard deviation) or was it a function of increasing compressive strength? This will affect the selection of statistical methods to analyze the data. Second, how can a statistically valid prediction be made if the in-place compressive strength of concrete from a correlation curve and a finite set of in-place pullout tests?

In designing a structural member to resist service loading, the engineer uses a strength property of concrete called the specified compressive strength f'_c. Because of the inherent variability of concrete in a structure, the contractor is required to use a concrete with an average strength well above the specified compressive strength (ACI 318,[4] ASTM C 94[5]). The current philosophy is to use concrete with an average strength such that not more than 10 percent of the concrete in the structure has a compressive strength less than the specified strength. Thus, the 10th percentile strength or the "characteristic strength," rather than the average strength, is used when assessing structural safety. To insure that safety during construction is comparable to safety during service conditions, one needs to know the in-place characteristic strength at the time of critical construction operations, such as formwork removal or post-tensioning.

No procedure has been agreed upon for determining the characteristic in-place strength based upon the results of in-place tests, such as the pullout test. However, one possible approach[6,7] is to directly convert the pullout strengths obtained from the field tests to equivalent compressive strengths by means of a relationship (correlation equation) that has been determined by regression analysis of previously generated data for the particular concrete being used at the construction site. The standard deviation of the converted data is then calculated. The characteristic compressive strength of the concrete is obtained by subtracting the standard deviation times a constant (which varies with the number of tests made and the desired level of confidence) from the mean of the converted data.

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ACI member William C. Stone is a research structural engineer with the Structures Division of the National Bureau of Standards. His current research interests include laboratory testing of full-scale bridge columns subjected to simulated seismic events and analytical computer studies on the three-dimensional behavior of framed structures during earthquakes. He holds degrees from Rensselaer Polytechnic Institute and the University of Texas at Austin.

ACI member Nicholas J. Carino is a research engineer in the Structures Division of the National Bureau of Standards. His current research interests include methods for measuring in-place strength of concrete and methods for locating defects in concrete. He is chairman of ACI Committee 306, Cold Weather Concreting, and serves as secretary of ACI Committee 228, Nondestructive Testing. He is a also a member of ACI Committee 437, Strength Evaluation of Structures. He is a graduate of Cornell University.

Charles P. Reeve is a consulting statistician in the Statistical Engineering Division at the National Bureau of Standards.

Table 1(a) — Summary of experimental data: Mixture proportions

Component	Percent absolute volume low-strength mix	Percent by weight low-strength mix	Percent absolute volume high-strength mix	Percent by weight high-strength mix
Type I cement	9.4	12.8	16.0	21.5
Coarse aggregate	40.3	47.0	40.3	46.3
Sand	28.1	31.8	21.5	24.0
Water	20.2	8.4	20.2	8.2
Air	2.0		2.0	

This approach assumes there is no error in the correlation and that the coefficient of variation of the in-place compressive strength is approximately equal to that of the measured pullout loads. However, a more statistically valid method of estimating concrete strength from in-place tests should make use of a correlation model that takes into account the errors associated with the two random variables. Also, and of equal importance, the minimum number of field pullout tests needed to characterize the concrete in a given placement with a desired level of confidence must be determined. These two issues address some of the last remaining barriers to the practical implementation of the pullout test (and other non-destructive methods) as a means of measuring the in-place strength of concrete.

RESEARCH SIGNIFICANCE

Nondestructive methods for determining the in-place strength of concrete are becoming increasingly popular owing to their simplicity, cost, and time effectiveness. However, until recently, no statistically valid method of predicting the in-place compressive strength from field-conducted nondestructive tests had been developed. Results of a series of pullout and companion cylinder tests that were conducted for the purposes of developing a generalized statistical analysis procedure for nondestructive tests are discussed. A simplified version of the detailed statistical method is presented.

EXPERIMENTAL PROCEDURE

The objective of the laboratory tests was to study the influence of apex angle and aggregate type on the nature of the relationship between pullout strength and cylinder compressive strength. Three aggregate types were used: river gravel, crushed limestone, and expanded shale lightweight. Apex angles of 54 and 70 deg were studied.

The pullout insert was identical to that detailed in Reference 3. This insert had dimensions that were based on those of a commercially available insert with one exception: the thickness of the embedded disk was increased to 12 mm (0.5 in.) to accommodate a displacement sensing rod, which would monitor the motion of the disk during testing. For testing efficiency, 11 inserts were placed into a 152 by 152 by 914 mm (6 by 6 by 36 in.) beam mold. The inserts were affixed at the mid-height of the form sidewalls to minimize the effect of top-to-bottom variations in strength. Inserts were alternately placed on opposite sides of the forms at 152-mm (6-in.) spacing.

There were four test series: Series I used 19 mm (¾ in.) nominal maximum-size river gravel and an apex angle of 70 deg; Series II used the same aggregate and a 54-deg apex angle: Series III used 19 mm (¾ in.) nominal maximum-size crushed limestone aggregate and a 70-deg apex angle; and Series IV used 19 mm (¾-in.) nominal maximum-size expanded shale lightweight aggregate and a 70 deg apex angle. A washed masonry sand, finer than that specified in ASTM C 33,[8] was used in the mixtures. This particular sand was used in previous NBS studies on the behavior of pullout tests in mortar. It's use here permitted comparison of the two test series. The coarse aggregates were screened with a 19-mm (¾-in.) sieve so that only particles smaller than 19 mm (¾-in.) were used in the mixtures. To achieve a wider variation in compressive strength without having to conduct many tests at early ages, two concrete mixtures of diffferent water-cement ratios were used. Each test series included a high- and low-strength concrete mixture, the proportions of which are presented in Table 1(a).

For each test series, four beams (each with 11 pullout inserts) and 20 cylinders (100 by 200 mm; 4 by 8 in.) were cast for both the low- and high-strength mixtures, except for Series I and II where only one set of cylinders was made for correlation with the pullout strengths for the two apex angles. Four by eight in. test cylinders were used because of the limited amount of available concrete; the mixes were prepared in a 4 ft^3 mixer. Five replicate cylinder tests were considered sufficient to determine the mean and standard deviation of compressive strength with at least the same degree of confidence as the 11 replicate pullout tests from a beam specimen (see later discussion in this paper).

At approximately one-day age the beam specimens and cylinders were removed from their molds and stored underwater in the laboratory. Prior to actual testing, thermal history measurements were performed on a trial mix to determine the maturity differences between the cylinders and the beams. The temperature measurements were performed with thermocouples placed in the centers of two cylinders and at the center and end of a beam specimen. Hourly average temperatures were recorded with a multichannel datalogger which operated at a five-minute scan rate. Prior to placement of the specimens in the water bath there was a minor difference in the peak temperatures as shown in Fig. 1. When the specimens were placed in the water bath there was a sudden drop in their temperatures and, thereafter, all temperatures were equal. The initial temperature differences resulted in about a 30 C-hr difference in maturity prior to placement in the water bath. However, this resulted in an inconsequential difference in the cumulative maturity values at the time of testing. Thus, it was concluded that the use of the water bath insured that both cylinders and beams developed the same maturity at the time of testing. The low-strength specimens were subsequently tested at ages of 1, 2, 8, and 28 days. High-strength specimens were tested at ages of 1, 3, 12, and 28 days.

The test apparatus and general testing procedure were identical to those described in Reference 3. Load was applied using a displacement-controlled servo-hydraulic system. This resulted in a maximum rate of loading of approximately 8.9 kN/minute. Failure of the specimen occurred within three minutes of the start of the test. A computer-controlled data acquisition system was used to determine the ultimate load. A load-versus-disk displacement plot was generated for each test.

Typically, six inserts were tested on one side of a beam, the beam was rotated, and the remaining five inserts on the opposite face were tested.

The procedure required about an hour during which the companion cylinder tests were also conducted. For test Series I and II, which were concerned with the effect of different apex angles, the above procedure varied slightly. To account for the effects of possible differences in concrete properties between specimens, half of each beam in this series (two beams were tested on each selected day) was tested with a reaction ring giving a 54-deg apex angle, while the opposite side was tested with a ring giving a 70-deg angle.

RESULTS

Appendix Table A gives individual test results for the 352 pullout tests and 120 companion cylinder tests conducted in this study. The identification (ID) numbers at the head of each column indicate the aggregate type, batch number, and test age, respectively. Table 1(b) gives averages of pullout force and cylinder strength, standard deviation, and coefficient of variation at each test age for each series. These are calculated from the data columns for each test presented in Table A1. In some of the pullout tests, radial cracking occurred and was often accompanied with a low pullout force. The Dixon test for outliers, as described in ASTM E 178-80[9] and in Reference 10, was used to discard low test results using a significance level of 0.05. The discarded results are identified in Table A with an asterisk.

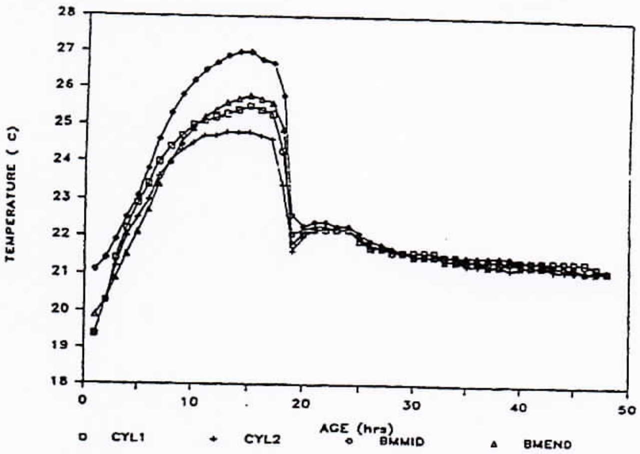

Fig. 1—Early age temperature histories of water-cured beam and cylinder test specimens

Fig. 2 and 3 plot the variabilities of the results as a function of averages. In Fig. 2(a) it is seen that the standard deviation for the cylinder tests tends to increase with increasing cylinder strength. This trend is consistent with previous observations on the variability of cylinder strengths in laboratory testing.[11] A similar, but more pronounced, variation in standard deviation was observed for the pullout tests, as seen in Fig. 2(b). By contrast, the coefficients of variation of both cylinder strength and pullout strength do not show any clear trends with increasing average strength, as indicated in Fig. 3. The average coefficient of variation for the cylinder tests was approximately 4 percent, which is consistent with the reported behavior of within-batch tests performed in the laboratory.[11] For the pullout tests with normal weight aggregates, the average coefficient of variation was about 10 percent, irrespective of the apex angle.

Two conclusions can be drawn from the variability of the pullout test data. First, the coefficient of variation for the lightweight aggregate tests ($CV = 6$ percent) is significantly lower than that for the river gravel and crushed limestone. This can be explained by the different failure mechanism attributed to these particular aggregates. The predominant failure mode for lightweight aggregate concrete is by propagation of the failure surface through the individual aggregates that happen to cross the failure surface, which runs from the insert disk edge to the inside edge of the reaction ring.[1,2,3] Because fracture occurs through the aggregates, pullout load is governed by the cement strength, and so ultimate behavior is similar to that exhibited by mortar, which is known[3] to have a significantly lower coefficient of variation for the pullout test than concrete. As previously reported,[3] pullout tests with mortar indicated a coefficient of variation of about 6 percent, which is similar to the present results with lightweight concrete. The harder aggregates, like river gravel and crushed limestone, on the other hand, typically do not fracture if the failure surface happens to intersect them. Instead, they must be pulled free from the bind-

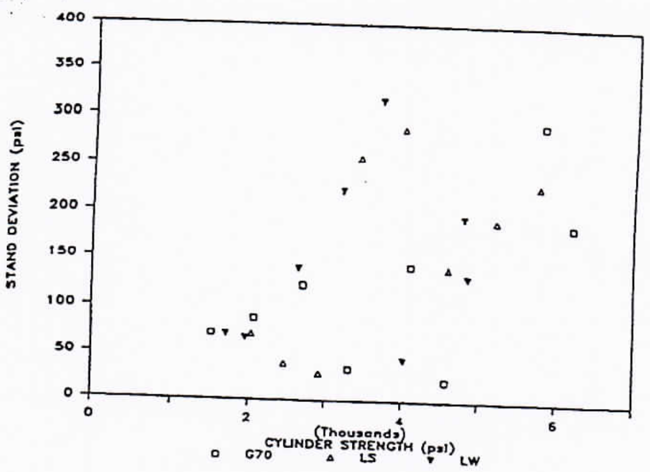

Fig. 2(a)—Standard deviation of cylinder compressive strength (5 replicates)

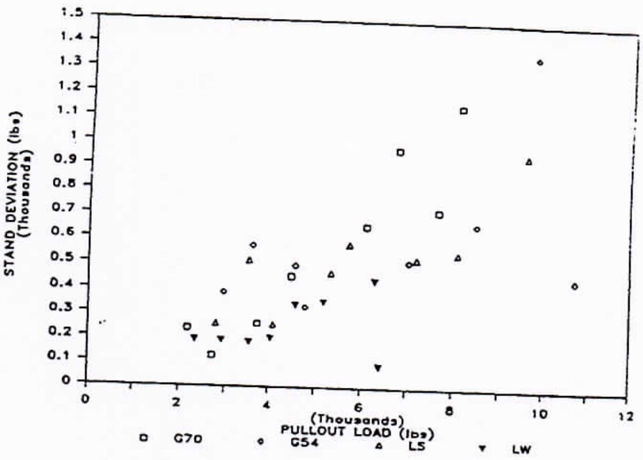

Fig. 2 (b)—Standard deviation of ultimate pullout load (11 replicates)

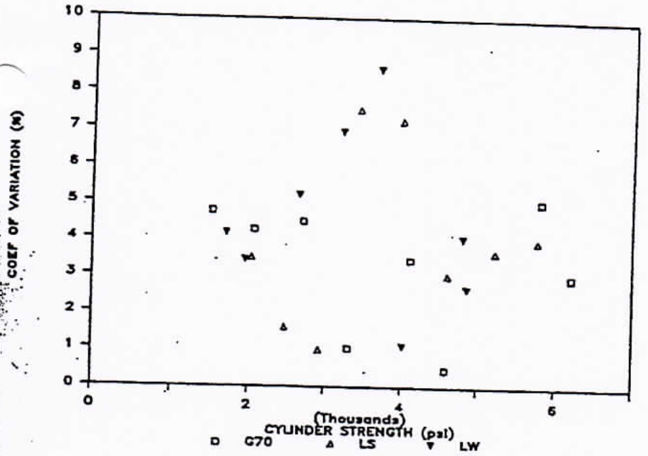

Fig. 3(a)—Coefficient of variation of cylinder compressive strength (5 replicates)

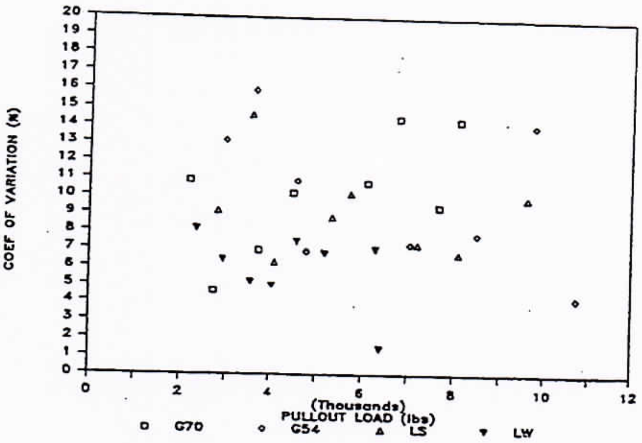

Fig. 3(b)—Coefficient of variation of ultimate pullout load (11 replicates)

has a coefficient of variation of 0.10 (expressed as a ratio) then the standard deviation of the logarithms of the data would be about 0.10.

Performing a linear regression of the natural logarithms of cylinder strength and pullout strength leads to the following equation for the correlation curve

$$\ln C = B_0 + B_1 \ln P \tag{1}$$

where

C = cylinder compressive strength
P = pullout strength
B_0 = intercept of the line
B_1 = slope of the line

By taking antilogarithms, an alternate form for Eq. (1) is the power function

$$C = AP^{B_1} \tag{2}$$

where

$$A = e^{B_0} \tag{3}$$

The curve passes through the origin, a logical requirement, since at the time of casting pullout strength and compressive strength are zero.

To deal with the problem of a large variability in the X-variable, that is, pullout strength, a more complex statistical method must be used to obtain the best unbiased estimators for B_0 and B_1. Reference 14 provides the procedure for this analysis, and this method has been used here.

The correlation curves are superimposed on the experimental data in Fig. 4 through 6. Also shown on these figures are the regression coefficients B_0 and B_1 and the estimated standard errors for these coefficients [$S(B_0)$ and $S(B_1)$]. In most cases the fit is nearly a straight line, as evidenced by B_1 values close to 1.0, the exception being for the crushed limestone, which exhibits some nonlinearity. For these tests, the limestone aggregate gave the least scatter about the best-fit line, while the river gravel aggregate and a 54-deg apex angle gave the most scatter.

Table 1(b) — Summary of experimental data: Average pullout loads and cylinder strengths

Series I & II: River gravel aggregate

I - 70 Deg			II - 54 Deg			I & II		
PO load, lb	S.D., lb	C.V., %	PO load, lb	S.D., lb	C.V., %	Cyl. Str., psi	S.D., psi	C.V., %
2185	238	10.9	2963	389	13.1	1510	72	4.8
2737	126	4.6	3603	573	15.9	2060	88	4.3
3734	260	7.0	4571	497	10.9	2710	122	4.5
4487	458	10.2	4788	331	6.9	3310	34	1.0
6097	658	10.8	7044	516	7.3	4110	142	3.5
6768	977	14.4	8491	669	7.9	4590	22	0.5
7660	724	9.5	9744	1368	14.0	5810	294	5.1
8091	1159	14.3	10728	457	4.3	6220	187	3.0

Series III: Crushed limestone

PO load, lb	S.D., lb	C.V. %	Cyl. Str. psi	S.D., psi	C.V. %
2806	257	9.2	2030	71	3.5
3535	515	14.6	2470	39	1.6
4091	257	6.3	2930	29	1.0
5346	473	8.8	3420	258	7.5
5740	585	10.2	3980	289	7.3
7207	529	7.3	4610	140	3.0
8103	553	6.8	5220	192	3.7
9595	957	10.0	5770	230	4.0

1000 lb = 4.45 kN
1000 psi = 6.89 MPa

Series IV: Lightweight aggregate

PO load, lb	S.D., lb	C.V., %	Cyl. Str., psi	S.D., psi	C.V., %
2339	191	8.2	1700	71	4.2
2935	189	6.4	1950	67	3.4
3552	184	5.2	2650	139	5.2
4043	201	5.0	3200	223	7.0
4576	342	7.5	3680	318	8.6
5187	354	6.8	4020	44	1.1
6293	445	7.1	4800	195	4.1
6413	92	1.4	4860	131	2.7

PO load = pullout load; S.D. = standard deviation; C.V. = coefficient of variation; Cyl. Str. = cylinder strength.

ing mortar matrix before ultimate failure occurs. This resistance arises from aggregate interlock, as described in References 1, 2, 3, and 12. The increased variability with harder aggregates arises from the random nature by which individual aggregates bridge the failure surface. The presence of a single large aggregate in the vicinity of the pullout insert, for example, could significantly raise the ultimate load for that particular test, and thus lead to a large within-test coefficient of variation. The second conclusion is that the coefficients of variation for the river gravel tests that utilized two different apex angles are nearly identical. This indicates that, as previously postulated in Reference 3, any apex angle within the range of 54 to 70 deg, which is specified in ASTM C 900,[18] will yield about the same coefficient of variation for the pullout test.

REGRESSION ANALYSIS

Current practice[6,7,13] for establishing a correlation curve between compressive cylinder strength and pullout force has been to average test data and use simple linear regression procedures such as found in elementary texts on statistics. When using such methods, two assumptions commonly are made:

1) The standard deviation of the dependent variable Y (concrete strength) is assumed to be constant throughout the range of Y values.

2) There is no uncertainty in the independent or "control" variable X (pullout strength).

The experimental data presented above show that these assumptions are not satisfied because standard deviation increases with increasing compressive strength both for cylinder and pullout tests, and the pullout strength is uncertain (in fact, it has a larger variability than cylinder strength).

A procedure for dealing with the case where the coefficient of variation, rather than standard deviation, of the data appears constant is to perform a linear regression on the logarithms of the data. It can be shown that if the coefficient of variation is constant, the standard deviation of the logarithms of the data is constant. Numerically, the standard deviation of the logarithms is approximately equal to the coefficient of variation of the data. For example, if a group of data

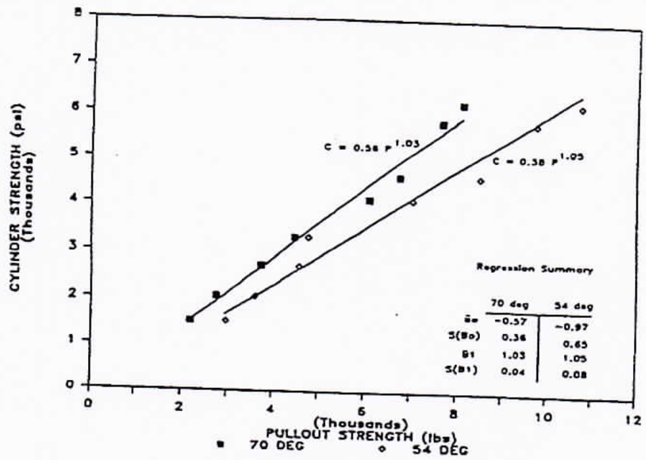

Fig. 4—Experimental data and regression lines for river gravel aggregate (apex angles of 54 and 70 deg)

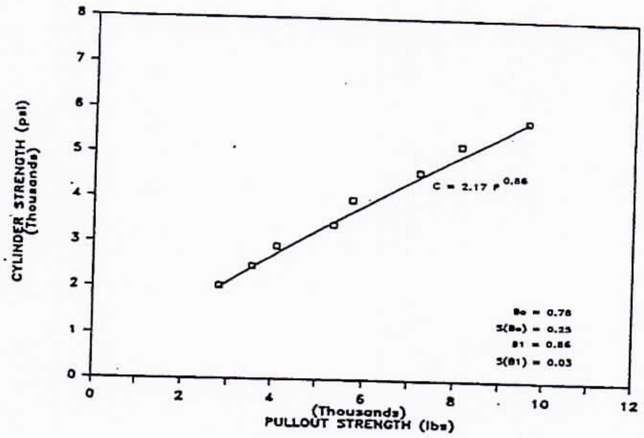

Fig. 5—Experimental data and regression line for crushed limestone aggregate (apex angle = 70 deg)

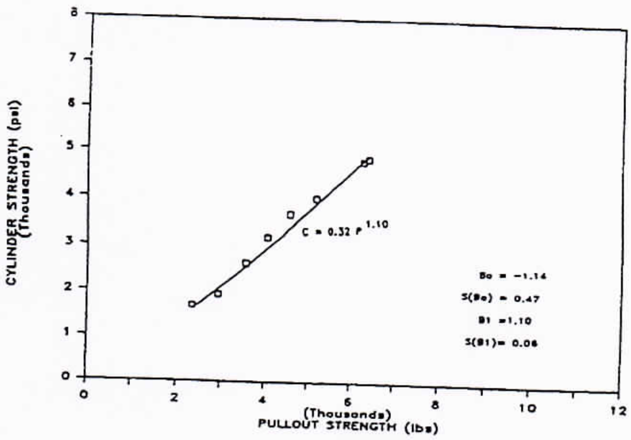

Fig. 6—Experimental data and regression line for lightweight aggregate (apex angle = 70 deg)

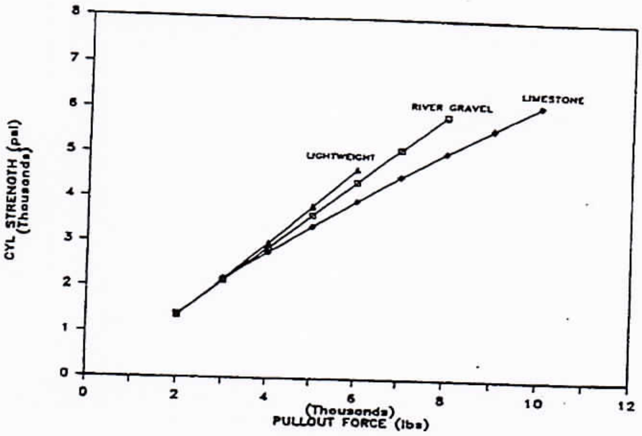

Fig. 7—Comparison of regression lines for river gravel, crushed limestone and lightweight aggregates (apex angle = 70 deg)

Fig. 7 compares the regression lines for the three different aggregate types tested with an apex angle of 70 deg. In a previous paper,[3] similar tests had indicated no significant difference in ultimate pullout force for differing aggregates. However, the previous tests were conducted at a single value of concrete compressive strength, approximately equal to 14 MPa (2 ksi). The current data show that for low compressive strength the pullout strengths are similar. However, beyond a compressive strength of about 14 MPa there is divergence in the three regression lines. It can be seen that for equal values of cylinder strength, the corresponding pullout strength is a function of aggregate type. Fig. 7 shows that, for equal cylinder strength, the concrete with river gravel aggregate has higher pullout strength than the concrete with lightweight aggregate. Based on statistical tests, it was shown that the three curves are significantly different. This offers direct proof that aggregate type affects the relation between compressive strength and pullout force.

IN-PLACE STRENGTH PREDICTION

To assess safety during construction we need to determine the in-place characteristic strength of the concrete and compare it with the strength required for structural safety. As mentioned earlier, the characteristic strength typically has been defined as that value of compressive strength that would be expected to be exceeded with 90 percent probability in the structure.

To estimate the in-place characteristic strength requires a relation between cylinder strength and pullout strength and information about the standard deviation of the in-place concrete strength. The correlation equation is used to estimate average cylinder strength, and the standard deviation is used to estimate the difference between average strength and the characteristic strength. However, there is error in the estimate of the average strength and there is error in the estimate of the standard deviation. Fig. 8(a) and (b) are schematic representations of the probability distributions for the average strength and the standard deviation. Because the characteristic strength is estimated from the estimates of average and standard deviation, it too will have error. Fig. 8(c) shows a schematic representation of the probability distribution of the characteristic strength. For structural safety, there should be a high probability that the true characteristic strength exceeds the required strength. The appropriate probability value depends on the consequences of a structural failure. For ordinary structures, a probability value of 75 percent has been suggested,[6] but higher values may be justified, for example, for structures prone to progressive collapse. Additional discussion of this subject is presented later in this paper.

If experimental data are directly available for in-place compressive strength, a one-sided tolerance limit approach can be used to determine the characteristic strength at any desired probability level. For concrete with reasonable quality control, a normal distribution can be assumed for the in-place data.[15] The characteristic strength can be determined as follows

$$C_{.10} = C_a - K\,SD_c \qquad (4)$$

where C_{10} = the estimated characteristic strength, i.e., the lower 10th percentile of strength (10 percent defect)
C_a = sample mean strength
K = one-sided tolerance factor
SD_c = sample standard deviation

The tolerance factor K depends on n (the number of tests) and the probability that the true characteristic strength exceeds the required strength. The method is presented in detail here since some have attempted to apply it directly to the prediction of in-place strength from pullout tests. While such an approach is applicable for predicting the characteristics compressive strength from cylinder tests, it is inappropriate for predicting the characteristic compressive strength from in-place pullout tests. This is so for two reasons. The method presented in References 6 and 7 converts the individual pullout strength values to equivalent compressive cylinder strengths by means of the correlation equation. The statistical analysis described by Eq. (4) is performed on the equivalent compressive strengths. This implies that the coefficient of variation of the equivalent compressive strengths is approximately equal to that of the in-place pullout forces, a fact that is clearly contradicted in all recent publications which compare pullout and cylinder tests.[3,7,13,16,17] For within-batch tests, cylinder strength typically has a coefficient of variation of approximately 5 percent or less, while pullout tests exhibit a coefficient of variation of about 10 percent. Secondly, a direct conversion of pullout force to an equivalent compressive strength, such as described in References 6 and 7, neglects the error in the regression line.

To account for the deficiencies in the tolerance limit approach, a procedure was developed for computing the characteristic strength for any desired confidence level. The details of the procedure are given in Reference 14, and only the key steps are given here. First, the variability of the in-place compressive strength is established based on the in-place pullout tests and the results from the correlation tests. It is assumed that the standard deviation of the in-place strength can be computed as follows

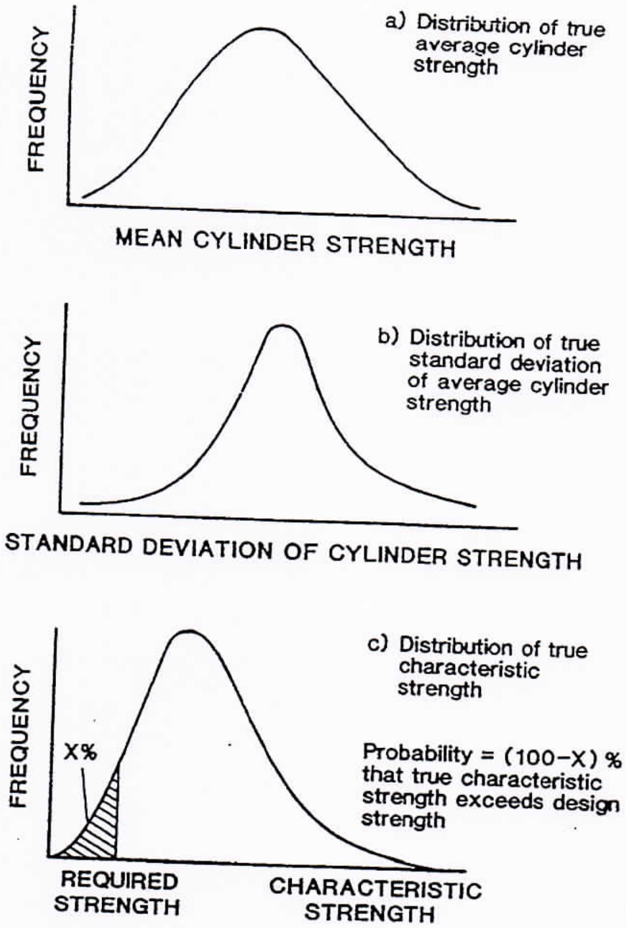

Fig. 8—Schematic illustrations of probability distributions for average cylinder strength, standard deviation of cylinder strength, and characteristic strength

$$S_{cf} = \frac{S_{cl}S_{pf}}{S_{pl}} \qquad (5)$$

where

S_{cf} = estimated standard deviation of the logarithm of in-place cylinder strength
S_{cl} = standard deviation of the logarithm of cylinder strength from correlation testing program
S_{pf} = standard deviation of the logarithm of in-place pullout strength, and
S_{pl} = standard deviation of the logarithm of pullout strength from correlation testing program

Eq. (5) is based on the assumption that the ratios of standard deviations of cylinder strength to pullout strength have the same value in the field as in the laboratory. This contrasts with the tolerance limit approach where the coefficient of variation of the in-place cylinder strength is assumed to equal that of the in-place pullout tests.

Based on the estimated standard deviation and estimated average value of in-place cylinder strength, the value of the characteristic strength that is, the 10th percentile strength can be estimated. The average in-place cylinder strength is obtained from the correlation equation using the average of the in-place pullout tests. Because the estimated cylinder strength is a random variable, the true cylinder strength will exceed the estimated value with only 50 percent probability. Therefore, it is necessary to compute the value of cylinder strength that is expected to be exceeded at some higher confidence level, i.e., the characteristic strength. This requires computing an estimate of the variance of the cylinder strength as described previously. To obtain the characteristic strength at the desired confidence level, the product of the square root of the variance of the cylinder strength and the Student t-value for the desired confidence level[14] are computed. This product is subtracted from the estimated value of cylinder strength to arrive at a characteristic strength value.

To compare the characteristic strengths based on tolerance limit approach given in Reference 6 and 7 and on the procedure described previously, a series of hypothetical in-place pullout tests were generated by computer simulation, and these values were used as input to calculate characteristic strength. Three nominal levels of average pullout load — 13, 22, and 36 kN (3, 5, and 8 kips) — and two levels of coefficient of variation in pullout load (10 and 20 percent) were used. Thus, for each correlation corresponding to the four test series, there were six different sets of hypothetical pullout test results. Each set contained 10 pullout loads. Simulations were not conducted for lightweight concrete at the 36 kN (8 kip) load level, since this is beyond the range of the experimental data. Characteristic strengths were calculated using the tolerance limit approach for confidence levels of 0.75 and 0.95. For the NBS approach,[14] confidence levels of 0.75, 0.95, and 0.99 were used.

The results of the calculations are summarized in Table 2. Column 1 identifies the test series, and Column 4 gives the average in-place cylinder strengths based on the appropriate correlation equation and the average pullout loads given in Column 2. Columns 5 through 9 give the characteristic strengths for the two methods and for different confidence levels. The columns labeled "NBS" refer to the approach described in Reference 14. For ease of comparison, the results are presented in graphical form in Fig. 9, which shows the differences between the characteristic strengths based on the two approaches. The differences between the values of characteristic strengths based on the tolerance limit approach and the values based on the NBS approach are expressed as a percentage of the NBS values.

It is seen that the tolerance limit approach results in characteristic strength values that are well below those based on the NBS approach. It is also seen that the difference is greater at the 0.95 confidence level than at the 0.75 level. Thus, it is concluded that the tolerance limit approach, in which the standard deviation of the in-place cylinder strength is assumed to equal that of the in-place pullout tests, results in very conservative estimates of characteristic strength compared to the NBS approach. Fig. 9(b) shows the percentage differences between characteristic strengths based on the tolerance limit approach at a confidence level of 0.75 and

Table 2—Comparison of estimated in-place characteristic strength

Test series (Col. 1)		In-Place pullout tests			Characteristic strength				
					75% prob.		95% prob.		99% prob.
		Avg. load, lb (Col. 2)	C.V., % (Col. 3)	Avg. str., psi (Col. 4)	NBS, psi (Col. 5)	Tol. lim., psi (Col. 6)	NBS, psi (Col. 7)	Tol. lim., psi (Col. 8)	NBS, psi (Col. 9)
I	(G70)	3010	10.9	2140	1980	1760	1880	1600	1810
I	(G70)	3030	20.7	2130	1850	1410	1700	1100	1590
I	(G70)	4970	11.9	3590	3290	2870	3130	2570	3000
I	(G70)	5060	20.3	3610	3130	2400	2890	1880	2700
I	(G70)	8120	9.9	5960	5520	4930	5260	4520	5070
I	(G70)	8110	21.9	5820	4870	3720	4400	2820	4060
II	(G54)	3010	10.9	1690	1530	1400	1420	1270	1330
II	(G54)	3030	20.7	1680	1430	1120	1300	870	1200
II	(G54)	4970	11.9	2860	2600	2290	2450	2050	2340
II	(G54)	5060	20.3	2880	2475	1910	2270	1490	2120
II	(G54)	8120	9.9	4790	4410	3960	4160	3630	3980
II	(G54)	8110	21.9	4680	3890	2980	3500	2250	3210
III	(LS)	3010	10.9	2170	1970	1840	1890	1700	1830
III	(LS)	3030	20.7	2160	1810	1540	1680	1270	1590
III	(LS)	4970	11.9	3350	3010	2780	2880	2540	2780
III	(LS)	5060	20.3	3370	2820	2400	2620	1990	2470
III	(LS)	8120	9.9	5120	4690	4390	4510	4090	4380
III	(LS)	8110	21.9	5020	4030	3460	3680	2780	3430
IV	(LW)	3010	10.9	2180	1860	1760	1760	1580	1680
IV	(LW)	3030	20.7	2160	1630	1380	1470	1040	1360
IV	(LW)	4970	11.9	3780	3180	2980	2990	2640	2850
IV	(LW)	5060	20.3	3810	2860	2450	2590	1860	2390

1000 lb = 4.45 kN.
1000 psi = 6.89 MPa.
C.V. = coefficient of variation; Tol. lim. = tolerance limit; prob. = probability.

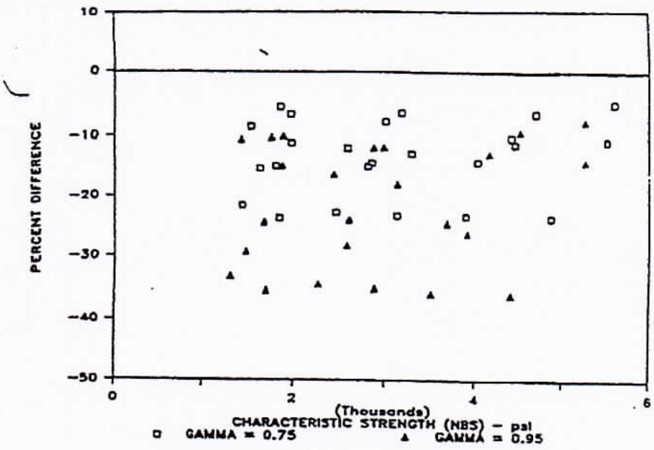

Fig. 9(a)—Percent difference in computed characteristic strength (tolerance factor method minus NBS method). Confidence level for tolerance factor method = 75 percent and 95 percent; for the NBS method = 95 percent.

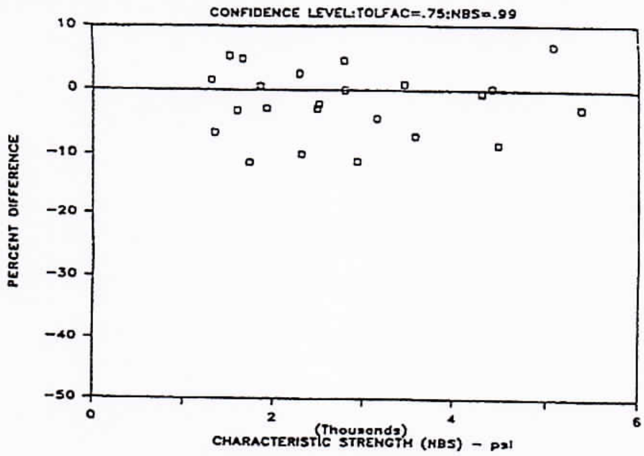

Fig. 9(b)—Percent difference in computed characteristic strength (tolerance factor method minus NBS method) for a confidence level of 75 percent for the tolerance factor method and 99 percent for the NBS method

ose computed at the 0.99 confidence level using the approach. This shows that what is implied to be a 0.75 confidence level with the tolerance limit approach is approximately a 0.99 confidence level with the NBS approach.

All of this serves to raise the question: What is the acceptable confidence level to be used in the evaluation of the characteristic strength? While it is beyond the scope of this paper to answer this question definitively, two factors that should influence the desired level of confidence can be identified. Each deals with the assessment of risk.

1) Basis of required strength criterion: Fast construction schedules, calling for one-day form removal, and early age post-tensioning are becoming common. Such early age operations are critical in the life of a structure, owing largely to the likelihood of unanticipated construction loads on an initially weak structure. There currently a "rule of thumb" which calls for an in- e strength of not less than ¾ f_c' prior to form re- val. Should such rule of thumb methods merit a higher degree of required confidence than a detailed structural analysis that accounts for construction loads?

2) The importance of the structure: The acceptable level of confidence will be directly proportional to the value of the structure and to the potential losses, both material and human, should a collapse occur. For a single-story structure, a lower confidence level may be more acceptable than for a high-rise building.

Actual recommended values for the suggested level of confidence will need to be arrived at through debate and discourse in the appropriate code committees.

REQUIRED NUMBER OF PULLOUT TESTS

How many individual in-place pullout tests must be performed for a given concrete placement so that a prescribed level of confidence results, in that the measured average pullout strength is representative of the true average in the structure? Presently, no recommendations exist in ASTM C 900[18] that parallel the requirements found in ASTM C 94[5] or ACI 301[19] for the number of cylinder tests to be performed for a specific volume or surface area of structural concrete.

There is some guidance available in ASTM E 122[20] (Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process). For any test, the number of required tests depends on 1) the coefficient of variation of the test; 2) the acceptable error between the sample average and the true average; and 3) the probability that the allowable error will be exceeded. As an equation, the required number of tests *(n)* is given as

$$n = (k\ V/e)^2 \qquad (6)$$

where

e = acceptable error between the sample average and the true average, expressed as a fraction of the true average

V = the prior estimate of the coefficient of variation of the test results, expressed as a fraction

k = a factor dependent on the probability that the acceptable error will be exceeded.

Eq. (6) can be used to find the ratio of the number of tests for two tests with different coefficients of variation. Assuming that the average test results represent the true averages with the same degree of confidence, the following equation results

$$n_1/n_2 = (V_1/V_2)^2 \qquad (7)$$

where

n_1/n_2 = ratio of the number of required tests
V_1/V_2 = ratio of the coefficients of variation

For example, if the ratio of coefficients of variation of two tests = 2, then the ratio of the required number of tests = 4.

Examination of laboratory data from NBS (approximately 1400 tests) and elsewhere[16] show that an approximate value for the coefficient of variation for the pullout test is 10 percent. For comparison purposes, cores can be assumed to provide the best measure of the in-place compressive strength. Published data on core testing appear to indicate that the approximate within-batch coefficient of variation for testing cores is 5 percent. Based on these values and Eq. (7), it can be concluded that the ratio of the number of pullout tests to the number of core tests should be about four. This provides assurance that the average pullout strength is known with the same degree of certainty as the average core strength.

To address the question of the actual number of pullout tests to characterize a given placement, the requirements of ACI 301,[19] dealing with sampling frequency for strength tests for acceptance of concrete can be used as a basis for extrapolation. The requirements are that samples be taken not less than once a day, nor less than once for every 76 m^3 of concrete, nor less than once for each 465 m^2 of surface area for slabs and walls. ASTM C 94[5] and ACI 318,[4] by contrast, require that samples be taken for every 115 m^3 of placed concrete. If one were willing to accept 2 or 3 individual cores at this sampling frequency, then one could accept 8 to 12 individual pullout tests as per Eq. (7). Note that this assures that the average pullout strength is known with the same degree of certainty as the average core strength. Predicting the in-place compressive strength from the pullout test still requires the use of a correlation equation such as that derived earlier in this paper and the appropriate statistics described in Reference.[14]

The preceding discussion supports the recommendations of Bickley,[7] Malhotra,[16] and Khoo[13] that an average of about 10 in-place pullout tests should be used to characterize a given concrete placement. However, from a practical point of view a contractor will need to place approximately 50 percent more than this amount in the event that the first five pullout tests indicate substantially understrength concrete. Ten tests would still be left for final verification after a suitable strength-gain period.

SUMMARY

Laboratory tests results showed that the within-batch standard deviation of ultimate strength for cylinder tests and pullout tests increased with strength over the range of compressive strengths tested. By contrast, the coefficients of variation for each type of test were constant over the range of compressive strengths and the average values were about 4 percent for cylinder tests and 10 percent for pullout tests where relatively hard coarse aggregates were used in the concrete. Tests conducted using apex angles of 54 and 70 deg indicated that both geometries produced nearly identical coefficients of variation. Pullout tests conducted in lightweight concrete exhibited coefficients of variation of only 6 percent, significantly lower than that for harder aggregates, and similar to the coefficient of variation for pullout tests in mortar reported in Reference 3. This difference is attributed to a change in the failure mode.

There are four required steps in using the pullout test (or other in-place tests) to predict the in-place strength of concrete. These are: 1) conducting a correlation series of cylinder and pullout tests at different ages for the particular concrete that will be used at the construction site; 2) development of a correlation equation that relates pullout force to compressive strength based on the tests conducted previously; 3) selection of the appropriate number of in-place tests; and 4) use of an appropriate statistical method to calculate the characteristic in-place compressive strength based on the in-place pullout tests.

When conducting the correlation tests it is essential that the concrete specimens used for the cylinder tests and for the pullout tests have the same maturity. Thermal measurements have shown that storing all specimens underwater immediately after form stripping leads to satisfaction of this criterion. The following minimum data requirements (although arbitrary) appear adequate for establishing the correlation equation: 6 strength levels that span the expected in-place strength to be measured at the construction site; 10 pullout tests; and 3 companion cylinder tests at each test age.

Because the standard deviation of the ultimate load increases with increasing compressive strength for both cylinder and pullout tests, linear regression must be performed on the natural logarithms of the pullout forces and cylinder strengths. This amounts to the fitting of a power function to the experimental data with an intercept through the origin. However, because there is uncertainty in X (pullout force) a more rigorous analysis than simple linear regression is needed to determine the best unbiased estimators of the regression coefficients and their variances. Such a procedure is detailed in Reference.[14]

The recommendations in ASTM E 122 led to a rational conclusion of the number of pullout tests needed to characterize the pullout strength of a given concrete placement. It is recommended that 8 to 12 pullout tests be performed for a given concrete placement. It is further recommended that approximately 12-18 pullout inserts be set per 76 m^3 (100 yd^3) to allow for the possibility of significantly understrength concrete being detected during the first few tests. This will leave a sufficient number of inserts for a meaningful test following an appropriate period of strength gain.

The tolerance factor method used to estimate the characteristic in-place compressive strength based on field-conducted pullout tests is incorrect for two reasons: 1) it does not account for the error in the fit of the regression line to the experimental correlation data; and 2) it assumes the in-place compressive strength has the same variability as the pullout test. The former reason can lead to unconservative results if there is considerable scatter in the experimental correlation data. The latter often leads to grossly conservative predictions of the characteristic strength, since cylinder strength is known to have a coefficient of variation less than half that of pullout strength. A rigorous statistical method was subsequently derived[14] as part of the NBS study which accounts for the error in the regression line, as well as the differences in in-place variance. The NBS method indicated as much as 40 percent conservatism in the tolerance factor approach, depending on the in-place coefficient of variation of the pullout tests.

Due to the complexity of implementing the NBS method for hand calculation two companion papers have been prepared* which permit the contractor to make use of personal computers to rapidly conduct the statistical analyses of correlation data and in-place tests. In the first paper the use of commercial "spreadsheet" programs was demonstrated; in the second an interactive FORTRAN program will be presented. The interactive program has the advantage of permitting the computer to prompt the inexperienced user for necessary data and issue advice on the interpretation of the results.

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REFERENCES

1. Stone, William C., and Carino, Nicholas J., "Deformation and Failure in Large-Scale Pullout Tests," ACI JOURNAL, *Proceedings* V. 80, No. 6, Nov.-Dec. 1983, pp. 501-513.

2. Stone, William C., and Carino, Nicholas J., "Comparison of Analytical and Experimental Internal Strain Distribution for the Pullout Test," ACI JOURNAL, *Proceedings* V. 81, No. 1, Jan.-Feb. 1984, pp. 3-12.

3. Stone, William C., and Giza, Bruce J., "The Effect of Geometry and Aggregate on the Reliability of the Pullout Test," *Concrete International: Design & Construction*, V. 7, No. 2, Feb. 1985, pp. 27-36.

4. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, De-, 1983, 111 pp.

5. "Standard Specification for Ready Mixed Concrete," (ASTM C 94-83), *1983 Annual Book of ASTM Standards*, V. 04.02, ASTM, Philadelphia, p. 64.

6. Hindo, Kal R., and Bergstrom, Wayne R., "Statistical Evaluation of the In-Place Compressive Strength of Concrete," *Concrete International: Design & Construction*, V. 70, No. 2, Feb. 1985, pp. 44-48.

7. Bickley, J. A., "The Evaluation of Acceptance of Concrete Quality by In-Place Testing," *In Situ/Nondestructive Testing of Concrete*, SP-82, American Concrete Institute, Detroit, 1984, pp. 95-109.

8. "Standard Specification for Concrete Aggregates," (ASTM C 33-82), *1983 Annual Book of ASTM Standards*, V. 04.02, ASTM, Philadelphia, pp. 12-21.

9. "Recommended Practice for Dealing with Outlying Observations," (ASTM E 178-80), *1986 Annual Book of ASTM Standards*, V. 14.02 ASTM, Philadelphia, pp. 138-163.

10. Natrella, Mary Gibbons, "Experimental Statistics," *Handbook* No. 91, U.S. Department of Commerce, National Bureau of Standards, Washington, D.C., 1963, p. 17-4.

11. Philleo, Robert E., "Increasing the Usefulness of ACI 214: Use of Standard Deviation and a Technique for Small Sample Sizes," *Concrete International; Design & Construction*, V. 3, No. 9, Sept. 1981, pp. 71-74.

12. Reinhardt, Hans W., and Walraven, Joost C., "Cracks in Concrete Subject to Shear," *Proceedings*, ASCE, V. 108, ST1, Jan. 1982, pp. 207-224.

13. Khoo, L. M., "Pullout Technique—An Additional Tool for In Situ Concrete Strength Determination," *In Situ/Nondestructive Testing of Concrete*, SP-82, American Concrete Institute, Detroit, 1984, pp. 143-159.

14. Stone, William C., and Reeve, Charles P., "A New Statistical Method for Prediction of Concrete Strength from In-Place Tests," *Cement, Concrete and Aggregates*, V. 8, No. 1, Summer 1986, pp. 3-12.

15. Mirza, Sher Ali; Hatzinikolas, Michael; and MacGregor, James G., "Statistical Descriptions of Strength of Concrete," *Proceedings*, ASCE, V. 105, ST6, June 1979, pp. 1021-1037.

16. Carette, G. G., and Malhotra, V. M., "In Situ Tests: Variability and Strength Prediction of Concrete at Early Ages," *In Situ/Nondestructive Testing of Concrete*, SP-82, American Concrete Institute, Detroit, 1984, pp. 111-141.

17. Vogt, W. L.; Beizai, V.; and Dilly, R. L., "In Situ Pullout Strength of Concrete with Inserts Embedded by 'Finger Placing,'" *In Situ/Nondestructive Testing of Concrete*, SP-82, American Concrete Institute, Detroit, 1984, pp. 161-175.

18. "Standard Test Method for Pullout Strength of Hardened Concrete," (ASTM C 900-82), *1983 Annual Book of ASTM Standards*, V. 04.02, ASTM, Philadelphia, pp. 582-585.

19. ACI Committee 301, "Specifications for Structural Concrete for Buildings (ACI 301-84)," American Concrete Institute, Detroit, 1984, 33 pp.

20. "Recommended Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process," (ASTM E 122), *1986 Annual Book of ASTM Standards*, V. 14.02, ASTM, Philadelphia, pp. 48-53.

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*Carino, N. J., and Stone, W. C., "Analysis of In-Place Test Data with Spreadsheet Software," presented at the 1985 Annual Convention, American Concrete Institute, Denver, CO, Mar. 24-29, 1985 (unpublished) and Stone, W. C., "NBS-INSITU: A General Purpose Personal Computer Program for Statistical Analysis of In-Place Test Data," to be submitted to the ACI JOURNAL.

Table A—Individual pullout and cylinder test results

Series I: River gravel aggregate — 70 deg

ID No.	1-1-1	1-3-1	1-1-2	1-3-3	1-2-8	1-4-12	1-2-28	1-4-28
Age (d)	1	1	2	3	8	12	28	28
Pullout load, lb								
1	2580	5772	2892	6392	3940	6647	4022	6971
2	1876	6110	2646	7702	3703	8679	4093	8177
3	2124	6387	2728	6705	3632	7385	3566	8820
4	1988	5227	2754	5388	3685	7351	4178	9027
5	2309	6341	2717	5904	3282	7530	4890	8983
6	2164	4621	2647	5213	3727	6498	4741	5411
7	2164	6663	2783	8268	4218	8057	4694	8163
8	2074	6466	2675	6987	3385	8804	4528	8421
9	2415	6179	2970	7683	3706	7989	4828	9091
10	2481	6830	2792	7330	3954	7846	5042	8872
11	1864	6476	2506	6871	3837	7473	4779	7064
Cylinder strength, psi								
1	1610	4090	2110	4580	2705	6070	3295	6335
2	1550	4060	1925	4605	2850	5490	3355	6040
3	1495	4135	2150	4615	2540	5530	3270	6370
4	1430	4330	2020	4560	2660	6120	3335	6375
5	1460	3940	2090	4600	2800	5830	3295	6000

Series II: River gravel aggregate — 54 deg

ID No.	1-1-1	1-3-1	1-1-2	1-3-3	1-2-8	1-4-12	1-2-28	1-4-28
Age (d)	1	1	2	3	8	12	28	28
Pullout load, lb								
1	2309	7043	3187	8740	4895	7800	5174	*5951
2	3055	7309	3547	8998	4094	10649	4421	11289
3	3400	7791	3348	8186	5327	9334	4842	10580
4	3292	6297	3550	7562	4977	8463	4180	10883
5	2502	7945	4007	8772	4214	9890	4486	11392
6	3431	7071	3485	9468	4889	7589	5306	10188
7	3062	6935	3295	8849	3983	10440	4832	10432
8	2614	6519	4420	8895	4797	10928	4720	11328
9	3352	6510	4744	7167	4902	11637	4832	10322
10	2815	6818	2776	8153	3767	11150	4818	10411
11	2763	7245	3273	8613	4438	9309	5056	10459

Series III: Limestone aggregate — 70 deg

ID No.	2-1-1	2-2-1	2-1-2	2-2-3	2-1-8	2-2-12	2-1-28	2-2-28
Age (d)	1	1	2	3	8	12	28	28
Pullout load, lb								
1	2409	4636	3777	*4565	4160	*5952	4823	8006
2	2675	6281	3271	6887	4228	7875	5437	9608
3	3024	5770	2623	7232	3666	8663	6225	9421
4	2850	5901	3111	6724	4541	8213	5457	9775
5	2699	5635	4644	7311	3903	9069	5373	9875
6	2470	5559	3290	6249	4399	7567	5495	8177
7	2873	5991	3635	7046	4138	8553	5820	9569
8	2796	6171	3435	7428	4114	7530	4502	10971
9	3227	5836	3542	8219	4038	8393	4923	10836
10	2693	6569	3571	7472	3746	7541	5241	10379
11	3150	4795	3986	7499	4064	7626	5505	8932
Cylinder strength, psi								
1	1990	3740	2500	4470	2930	5235	3030	6020
2	2005	3720	2495	4655	2950	5315	3315	5815
3	2125	3945	2415	4525	2950	5460	3485	5790
4	2090	4425	2495	4829	2880	5170	3670	5845
5	1955	4075	2440	4565	2925	4940	3610	5395

Series IV: Lightweight aggregate — 70 deg

ID No.	3-1-1	3-2-1	3-1-2	3-2-3	3-1-8	3-2-15	3-1-28	3-2-84
Age (d)	1	1	2	3	8	15	28	28
Pullout load, lb								
1	1998	4397	2588	4842	3761	6415	4130	6281
2	2667	4412	2849	4997	3524	5788	4164	6468
3	2357	4322	3134	4958	3651	6672	3890	6489
4	2170	5043	2812	5121	3382	6952	3567	6378
5	2259	4773	3198	5760	3485	6592	4069	*5211
6	2540	4562	2915	5277	*2618	*3826	4278	6352
7	2491	5208	3028	5580	3248	5524	4259	6468
8	2351	4657	3103	5464	3513	6000	4040	6315
9	2441	4285	2800	4555	3850	6383	4004	6403
10	2290	4033	3084	5437	3687	6023	4159	*5862
11	2164	4639	2773	5071	3422	6582	3916	6565
Cylinder strength, psi								
1	1650	3425	1845	3945	2760	4915	3095	4925
2	1710	4060	1920	4025	2500	4775	3285	4900
3	1735	3900	2015	4025	2670	4775	2910	4860
4	1610	3300	1995	4065	2815	5030	3510	4980
5	1790	3720	1950	4025	2525	4510	3190	4640

*Outliers not considered in data analysis.
1000 lb = 4.45 kN.
1000 psi = 6.89 MPa.