Lok-tests determine the compressive strength of concrete

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Introduction

The lok-test is a test where a solid of revolution is extracted from the concrete by means of an embedded disc which is pulled out under application of a counterpressure, see Figure 1.

As described by Kierkegaard-Hansen [1] such tests can be used to determine the compressive strength of the concrete, as experiments have shown a linear relationship between pull-out force and concrete strength. At a first glance, one might expect the pull-out force to be dependent upon the tensile strength of the concrete. Below we shall show that the theory of plasticity for concrete does indeed predict a direct relationship between pull-out force and compressive strength, provided the geometry of the test equipment is chosen appropriately.

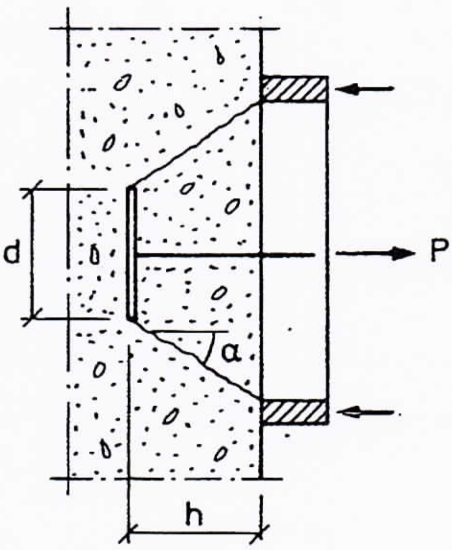

Figure 1. Sketch of lok-test.

Failure criterion

Consider Coulomb's well-known criterion for sliding failure

$$[\tau] = c - \sigma \tan\varphi \qquad (1)$$

where τ and σ are the values of shear and normal stress on the failure surface. The cohesion c and the angle of friction φ are material constants. In a coordinate system with axes σ and τ, equation (1) represent a straight line which is the boundary (Mohr's failure envelope) of the Mohr's circles corresponding to all possible stress states.

The failure criterion is shown in Figure 2 with two Mohr's circles corresponding to uniaxial compressive failure and uniaxial tensile failure.

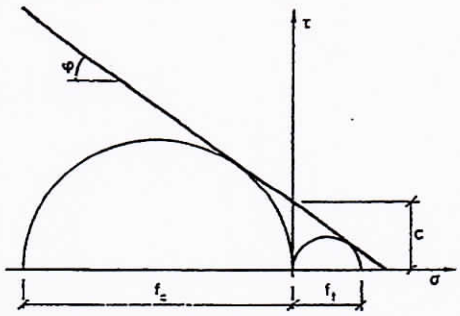

Figure 2. Coulomb's failure criterion.

From the figure we find

$$c = \frac{f_c\,(1 - \sin\varphi)}{2\cos\varphi} \qquad (2)$$

The sliding criterion is not sufficient to give a realistic picture of concrete failure. The uniaxial tensile strength obtained experimentally is inferior to the value suggested by Figure 2, and the tension specimens fail by separation rather than by sliding. It is therefore necessary to modify Coulomb's criterion adding the separation condition:

$$\sigma = f_t \qquad (3)$$

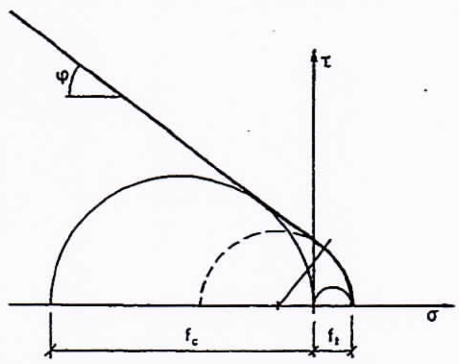

Figure 3. Coulomb's modified failure criterion.

Coulomb's modified failure criterion is shown in Figure 3. The dotted Mohr's circle corresponds to simultaneous failure by sliding and separation.

For concrete it has been shown that the angle at friction φ can be taken as 37°. If we suppose φ to be a constant, the straight line in the failure criterion is determined by one parameter – the compressive strength f_c as it is seen in (2).

Kinematical discontinuity

We now consider displacements in a kinematical discontinuity (failure surface) with the relative displacement rate V at the angle α with the discontinuity, see Figure 4.

The state of deformation is plane, and the concentrated strain rates in the coordinate system shown, are

$$\varepsilon_n = V \sin\alpha,\ \varepsilon_t = 0,\ \gamma_{nt} = V \cos\alpha \qquad (4)$$

Theory of plasticity for concrete

The modified Coulomb failure criterion is used as yield criterion for concrete. As yield law the normality condition is adopted.

At plane strain rate, the normality condition requires that the strain rate vector (4) be a normal to the yield criterion (Fig. 3), if we superimpose strain rate axes ε_n and γ_{nt} upon the stress axes σ and τ, as shown in Figure 5.

From the figure we note that we must have $\alpha \geq \varphi$ and we deduce that for $\alpha = \varphi$ the stress point is on the straight line of the yield locus. This means that the stress state in the failure surface depend on the compressive strength only. For $\varphi < \alpha < \pi/2$ the stress point is on the circular cut-off of the yield locus and the stress state depend on both the compressive and the tensile strength. The situation $\alpha = \pi/2$ corresponds to separation failure, depending upon tensile strength only. Returning to Figure 1 we note that if $\alpha = \varphi$ the pull-out force only depend on the compressive strength.

Kinematically discontinuities can be used to calculate upper-bound solutions for load-carrying capacities by means of the work equation. To do this we have to find the dissipation in the kinematical discontinuity. It can be shown to be

$$D = V \cdot \frac{1 - \sin\varphi}{2} f_c \qquad \alpha = \varphi \quad (5)$$

$$D = V \left(\frac{1 - \sin\alpha}{2} f_c + \frac{\sin\alpha - \sin\varphi}{1 - \sin\varphi} f_t \right) \quad \alpha \geq \varphi \quad (6)$$

For a more thorough discussion of the kinematical discontinuity and the corresponding dissipation see Jensen [2].

The pull-out force
An upper bound for the pull-out force of the embedded disc in Figure 6 can be determined assuming a failure surface with the shape of a truncated cone.

The area of the failure surface is

$$\frac{\pi h\,(d + h\,\tan\alpha)}{\cos\alpha} \qquad (7)$$

means of (6) and (7) the rate of internal work W_I is found to be:

$$W_I = V\left(\frac{1-\sin\alpha}{2} f_c + \frac{\sin\alpha - \sin\varphi}{1-\sin\varphi} f_t\right) \cdot \frac{\pi h\,(d + h\,\tan\alpha)}{\cos\alpha} \qquad (8)$$

The rate of external work W_E is

$$W_E = P \cdot V$$

Putting $W_I = W_E$ we get an upper-bound solution for the pull-out force P. With the normal values of φ and f_t/f_c and with not too small values of h/d (h/d > 1/5 if $f_c/f_t = 10$ and h/d > 1 if $f_c/f_t = 15$) the minimum is found for φ to

$$\frac{\pi h\,(d \cos\varphi + h \sin\varphi)\,(1 - \sin\varphi)}{2\cos^2\varphi} \qquad (10)$$

The failure mechanism of Figure 6 is not the most critical, however. A lower upper-bound may be obtained assuming a failure surface with a meridian curve which is not linear. Tests [1], [3] also show that the failure surface is not a truncated cone but a surface of revolution with a trumpet-like shape, as suggested by the dotted curve on Figure 6.

Lok-test
We can fix the failure mechanism to be a truncated cone by using a counterpressure as shown on Figure 1 if we choose $\alpha \cong \varphi$.
In the lok-test described by Kierkegaard-Hansen [1] we have h = d = 2,5 cm, and $\tan\alpha = \frac{3}{5}$ (α = 31°). If we suppose $\alpha = \varphi$ equation (10) yields:

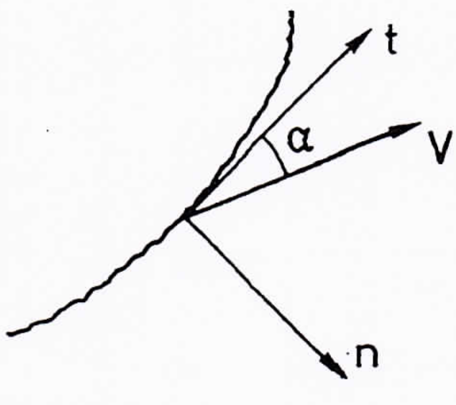

Figure 4. Kinematical discontinuity.

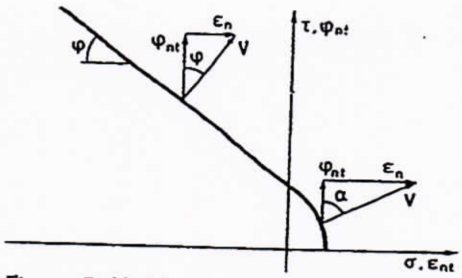

Figure 5. Yield criterion with displacement rate vectors.

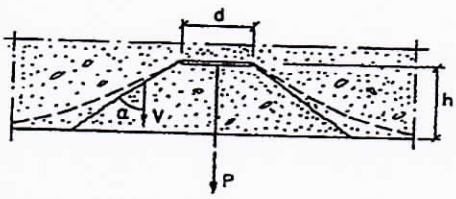

Figure 6. Embedded disc with a failure mechanism.

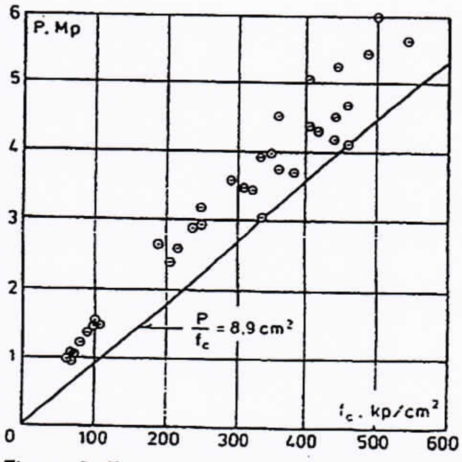

Figure 7. Tests from [4].

$$\frac{P}{f_c} = 8{,}9\ \text{cm}^2 \qquad (11)$$

It is now clear from these theoretical considerations that the pull-out force is a linear function of the compressive strength.

On Figure 7 the theoretical formula (11) is compared with tests carried out at the Structural Research Laboratory of the Technical University of Denmark [4].

We note that the theory underestimates the lok-strength, although the inclination of the line is satisfactory. At the testing, care has been taken to avoid adhesion between the disc and the concrete, hence such effects cannot account for the additional strength observed. Indeed, the maximum adhesion is determined by the tensile strength of the concrete, and this is far from sufficient to explain the difference. However, the fact that the experimental lok-strengths are greater than predicted by equation (11) is not surprising. The failure angle $\alpha = 31°$ imposed by the lok-test equipment is smaller than the angle of friction $\varphi = 37°$ usually assumed for concrete. According to the Coulomb criterion, Figure 3, failure surfaces with $\alpha < \varphi$ are impossible. Investigations have shown, however, that the failure envelope is not a straight line, but sligthly curved as indicated on Figure 8. Thus when $\alpha < \varphi$, the failure is not governed by the uniaxial compressive strength f_c but by the greater apparent strength f_c^* (cf. Figure 8). However, with a value α close to φ, we would expect a good correlation between lok-strength and cylinder strength, as confirmed by Figure 7.

Conclusion
Plastic analysis may be applied to determine the load-carrying capacity of a concrete embedded bolt which is pulled out under application of a counterpressure (lok-test). It is shown that when the angle between the direction of deformation and the failure surface is equal to the angle of friction for the concrete, then the pull-out force is proportional to the concrete compressive strength.

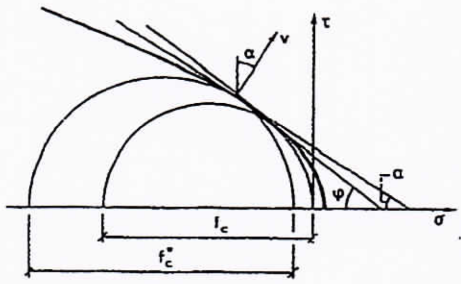

Figure 8. Failure envelope.

Acknowledgements

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