ULTRASONIC TOMOGRAPHY TECHNIQUE FOR EVALUATION CONCRETE PAVEMENTS

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CORE-FREE ULTRASONIC TECHNIQUE FOR EVALUATION CONCRETE PAVEMENTS

ABSTRACT

Ultrasonic tomography is an emerging technology that shows promise for quality assurance/quality control (QA/QC) during construction, or for making rehabilitation decisions in concrete pavements. However, the benefits of this emerging technology have not yet been fully captured for widespread use in highway infrastructure management. Verification of a state-of-the-art ultrasonic tomography device (MIRA) is presented through multiple field trials involving typical pavement constructability and rehabilitation issues. Field trials indicate that, while the device is a useful portable pavement diagnostic alternative capable of consistent thickness measurement, reinforcement location, and distress evaluation, significant efforts and user expertise are required for measurement and data interpretation of large scale application. Thus, software was developed for a more productive, objective signal interpretation method with automated analysis of reinforcement location in continuously reinforced concrete pavement. This type of automation for multiple applications shows promise for the use of ultrasonic tomography to improve large scale pavement QA/QC and rehabilitation projects in the future. Nevertheless, the research in the paper shows ultrasonic tomography to be an accurate, reliable, and convenient alternative or supplement to traditional techniques that can be utilized for a wide variety of small scale pavement diagnostics applications.

INTRODUCTION

Non Destructive Testing (NDT) techniques have been used to evaluate concrete pavements for many years. Some commonly used devices include the Ground Penetrating Radar (GPR), MIT SCAN 2 based on magnetic tomography for accurate dowel location, chaining, and seismic methods This paper explores a state of the art ultrasonic tomography device (MIRA) for use in evaluating concrete pavements.

Ultrasonic testing uses high frequency (greater than 20,000 Hz) sound waves to characterize the properties of materials or detect their defects. Sound waves are generated by transducers, travel through the material, and are received at the surface. Analysis of the signals by the receiving transducer provides information about the media through which the signal has propagated. Ultrasonic techniques have been successfully used in medical applications and flaw analysis in metals and composite materials for many years (Qi. et al, 2009; Sauvik et al. 2007). However, earlier applications of these types of ultrasonic technology for evaluation of Portland cement concrete (PCC) and asphalt structures have experienced difficulties. Traditional ultrasonic methods relied on time consuming liquid coupling and could not achieve the necessary penetration depths in pavement materials due to the heterogeneity causing excessive attenuation of the wavefront (Schubert and Köhler, 2001). These difficulties led to the use of rudimentary acoustic methods in pavement applications such as chain dragging or impact echo (IE). Conventional IE has experienced inconsistent results when applied to heterogeneous mediums such as concrete with only one mechanical impact that is highly dependent on variable duration (Schubert and Köhler, 2001; Carino, 2001). On the other hand, the dry point contact (DPC) transducers used in ultrasonic tomography have been developed with the capability of transmitting of relatively low frequency (55 khz) elastic waves that penetrate greater depths (Nesvijski, 1997; Mayer et al., 2008). Use of these transducers has been advanced and successfully applied for years in dealing with detailed evaluation of civil structures in Germany (Schubert and Köhler, 2001; Mayer et al., 2008; Khazanovich et al. 2005; Langenberg, K.J. et al., 2001; Marklein, R. et al. 2002).

These developments lead to the development of MIRA, the state-of-the-art ultrasonic tomography device for diagnostics of concrete structures, which utilizes the same principles that have been successful in medical and metal applications (ACSYS, 2008). MIRA utilizes 45 transmitting and receiving transducer pair measurements (see figure 1) in a less than 3 second scan resulting in a 2D depth profile (b-scan). The DPC transducers provide the necessary consistency of impact and wavefront penetration for diagnositcs up to 3 ft. deep. The multiple sensor pairs in each scan allows for the required redundancy of information to evaluate heterogeneous mediums such as PCC or asphalt. Figure 1 shows a manual MIRA measurement on the left in which the transducers are placed flush to the surface for a b-scan measurement. A b-scan is a 2 dimensional reconstruction of the concrete directly below where the scan was taken with high intensity of reflection areas indicating changes in acoustic impedance. On the right side of figure 1 the increased redundancy of information of MIRA (bottom) over conventional IE (top) can be observed, where the multi-static array of transmitting and receiving transducers creates 45 measurement angle pairs compared to one measurement pair in traditional IE.

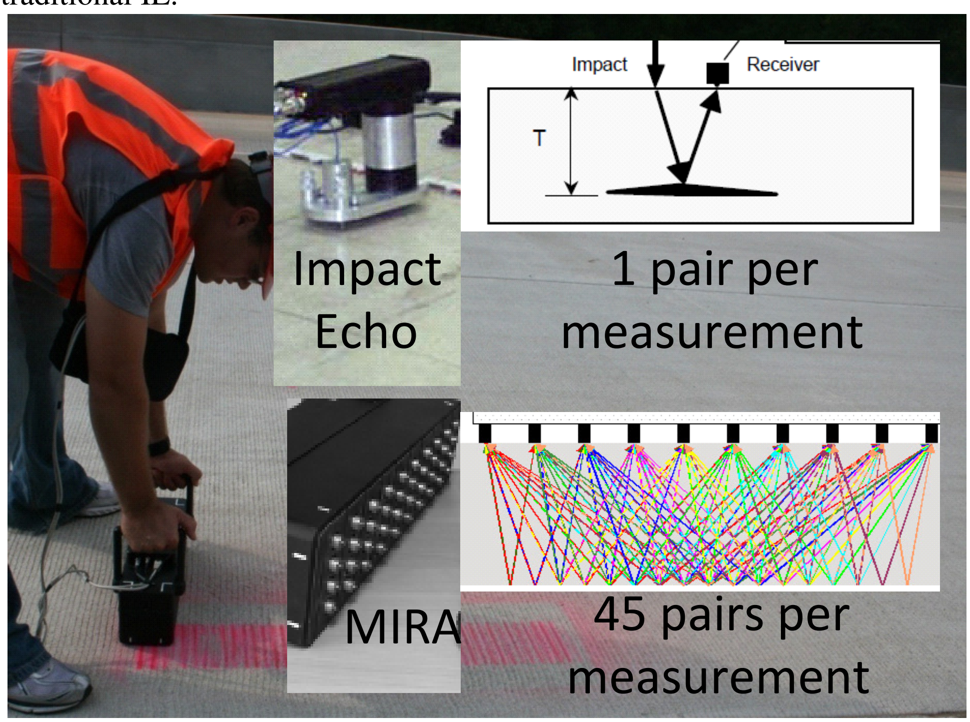

Figure 1. MIRA ultrasonic pitch-catch device and comparison with traditional impact echo method (Carino, 2001; ACSYS 2008).

MEASUREMENT PROCESS AND SIGNAL INTERPRETATION

The ultrasonic tomography device (MIRA), manufactured by Acoustic Control Systems, contains 40 "touch and go" transmitting and receiving DPC transducers (see Figure 1). The probes are firmly fixed in an array with 10 channels of 4 transducers resulting in a scanning aperture of 400 mm by 50 mm. Each of the probes can act as either receiver or transmitter with a default operation ultrasonic frequency of the 55 kHz (ACSYS, 2008). The device measures time of signal propagation between the transducers at fixed distances for material velocity calibration, and uses the synthetic aperture focusing technique (SAFT) for analysis in scan mode to reconstruct the medium below the measurement based on the shear wave reflections. SAFT has been found to be a feasible algorithm for use with the ultrasonic pitch-catch technology as well as other applications. The basis of the SAFT algorithm is given in equation 1 (Langenberg et al., 2001):

$$p_s(\underline{R}_0, t) \sim \frac{F\left(t - \frac{2R_0}{c}\right)}{R_0^2} \tag{1}$$

Where p_s is the scattered field; $\underline{R}_0$ is the location of the point source transducer on measurement plane (S_M) creating the incident field (p_i); F is the frequency spectrum of the transducer, t is the time after the transmission of the pulse, R_0 is the distance from the point source transducer, and c is the velocity of the pulse. It is assumed that the measured signal is a replica of the transmitted pulse shape, and that all point scatterers at the same radius from the point source yield the same "measured" signal.

The basic reconstruction procedure involves (Langenberg 2001 et al.; Marklein, 2002):

1) Assign the "measured" intensity at every depth in the A-scan from each measurement to all locations in the 180 degree fan around the point source
2) Apply step 1 for all point sources on the measurement surface
3) Use a superposition of the intensity of each A-scan that results in intersection points from the different measurement locations.
4) Focus the scatterers by considering these intersections to be the locations of the anomalies.

Each MIRA scan creates a two-dimensional display of echo intensity versus depth along the aperture. The positions with high intensity (red) are areas where there is a change in acoustic impedance, which could be an inclusion, interface between two layers, or damage to the concrete. A brief explanation of a typical signal with multiple 0.75 in. reinforcement bars in concrete pavement is shown in figure 2. A high intensity reflection can be observed at the three longitudinal reinforcement bar locations where the center of objects shown in the scans corresponds to the center of the longitudinal bars. To compute concrete cover, the radius of the bar (0.375 in.) should be subtracted from the depth of the center of the scatterer. A reflection can also be observed at a depth of about 12 in. at the concrete and base interface. It should be noted the reflection from the PCC/base interface does not extend throughout the horizontal range of the scan as it might be expected for such a continuous interface. This phenomenon is not indicative of any deterioration or lack of continuity, but rather a result of the limited aperture of the scan. As can be observed in figure 1, the 45 transmitting and receiving transducer pair measurements within a MIRA scan are concentrated toward the horizontal center. Thus, there is a higher redundancy and intensity of reflection (and thus more reliable information) towards the center of each scan. The thickness measurements can be made by applying the same method to the oblong reflection at the base as was used for the reinforcement measurements.

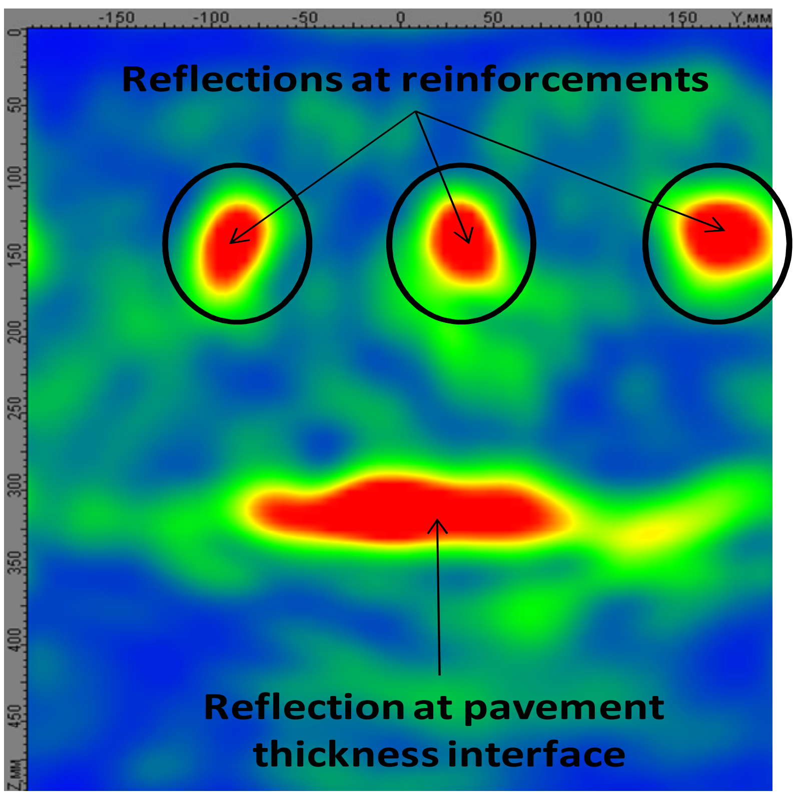

Figure 2. Examples of scans with reinforcement and PCC bottom surface reflections.

MIRA has proven capable of getting a consistent acoustic transmission and reception for various material and surface properties, including different tines and rough surfaces. The small size and low weight makes MIRA a very portable and flexible option for concrete pavement diagnostics.

FIELD TRIALS

Reinforcement Depth and PCC Thickness Measurement

The first field application of MIRA for evaluation of a continuously reinforced concrete pavement (CRCP) was conducted in the fall of 2009 on I-85 near Atlanta, GA. This project was suspected to have variability in concrete thickness and rebar depth. A magnetic based nondestructive method (pachometer) was used unsuccessfully for this application prior. Subsequent MIRA testing of over 3 miles showed that MIRA can reliably determine the depth of rebars with an accuracy significantly exceeding pachometer results. Figure 3 shows a comparison of pachometer concrete cover versus core measured results. It can be observed that a very low correlation was found (R^2 = 0.186). Figure 4 shows MIRA measured versus core measured concrete cover at seven locations. It can be observed that there is a high correlation (R^2 = 0.991) and accuracy (slope = 0.972) in the MIRA measurements with all measurements within the uncertainty of the core measurements (0.125 in).

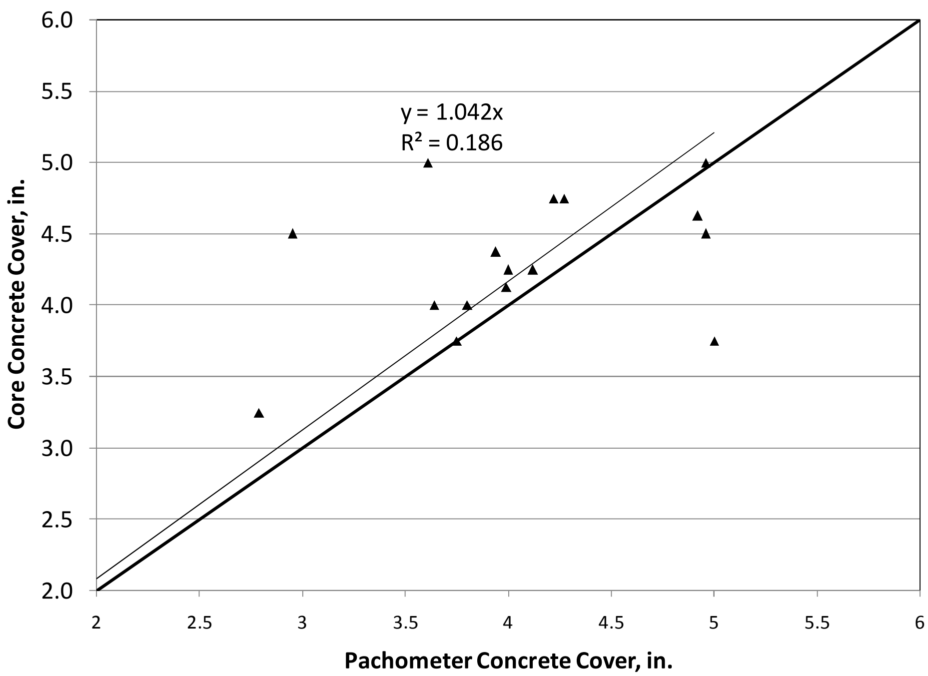

Figure 3. Magnetic device measurements versus core measurements of concrete cover.

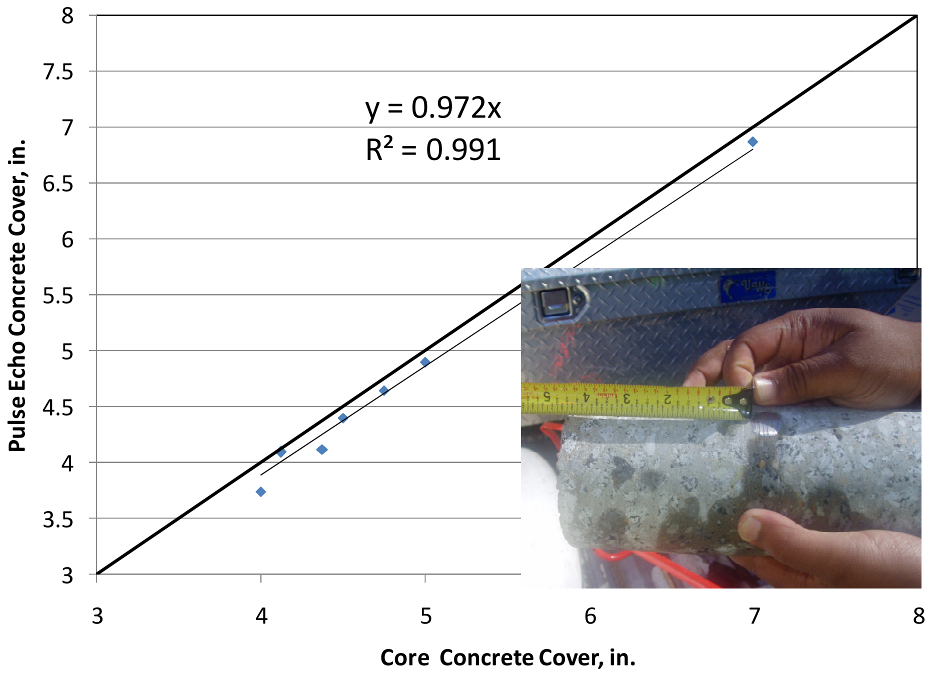

Figure 4. MIRA versus core measured concrete cover at 7 locations.

MIRA was also used for thickness determination in this study. Figure 4 shows MIRA measured versus core measured concrete cover at seven locations. While, multiple reflections and shadowing from the reinforcement, make the thickness measurements less precise for CRCP measurements compared to unreinforced concrete, a good agreement (R^2 = 0.967) and accuracy (slope = 0.971) can be observed for the same 7 core locations for thickness verification.

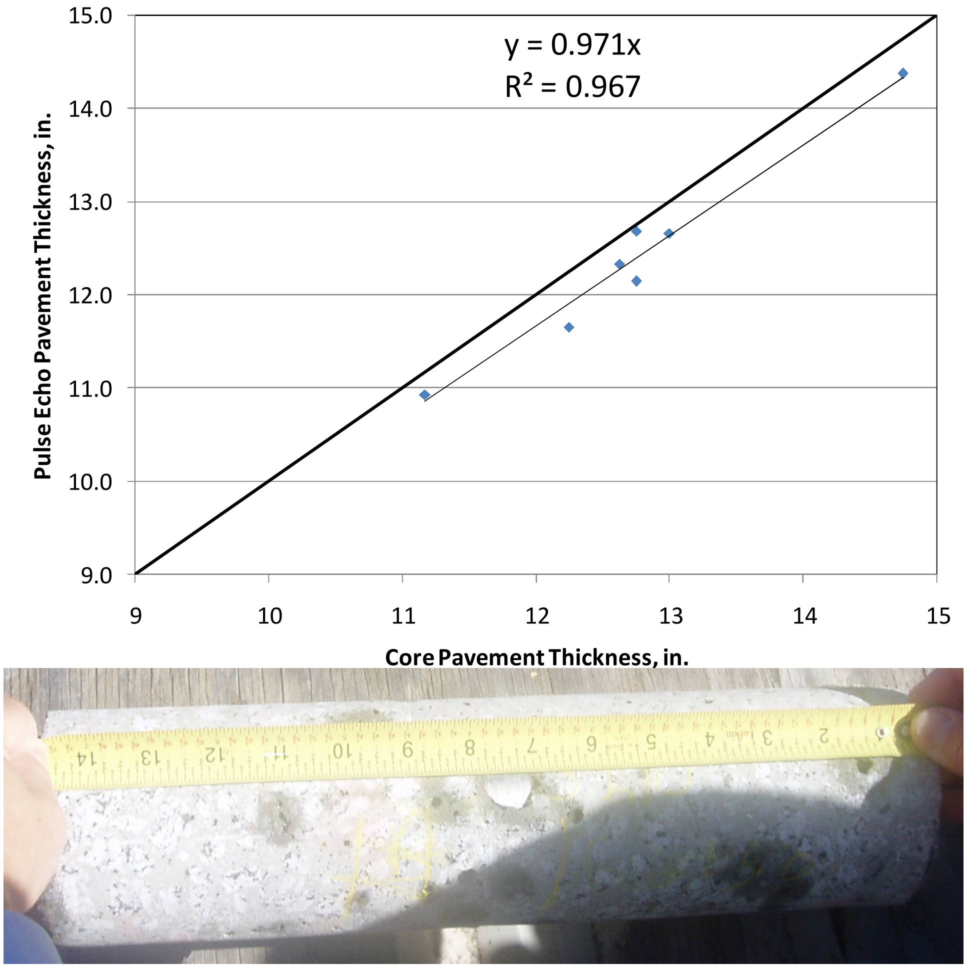

Figure 5. MIRA versus core measured concrete thickness at 7 locations.

With the core verification, large scale measurements could then be made, giving both concrete cover and thickness information 18 in. from the fog line every 50 ft along 3 miles of pavement. It should be noted that more cores should be conducted for a statistically sound verification. However, since it was an in-service pavement, the amount of cores that the Georgia Department of Transportation would allow was limited. Figure 6 shows the an example approximately 1 mile section on southbound I-85 that indicated concrete cover and pavement thickness well below the specifications with 67 percent of the measured concrete covers out of the range of 3.5 and 4.25 in (tolerance levels), and 18% percent below 2.5 inches of concrete cover. Overall, concrete slab thickness showed a similar out-of-tolerance pattern with 40 percent of the measurements outside the 11 to 13 inch thickness range.

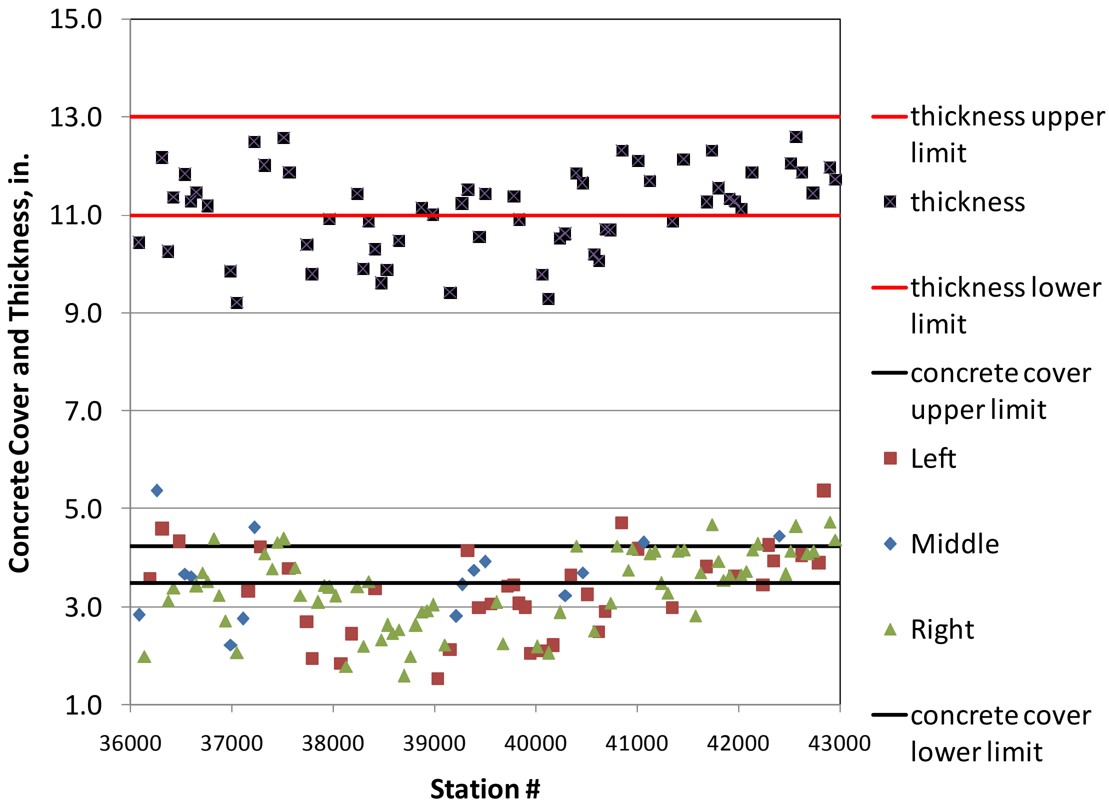

Figure 6. Southbound I-85 concrete cover and thickness MIRA measurements.

PCC Joint Assessment

A field trial was conducted at the Minnesota Road Research Facility (MnROAD) to verify the potential of MIRA for use in concrete pavement flaw detection. The information gathered by MIRA was used to help MnDOT personnel decide on coring and trenching locations where ultrasonic tomography scans indicated distress. Forensics showed MIRA to be capable of diagnosing presence and extent of deteriorating concrete and delamination near transverse joints.

The initial screening pinpointed some joint locations where the outputs signaled a high probability of deteriorating joints. Joint 5 in Cell 11 is a good example to illustrate the capabilities of MIRA, as it contained portions suspected to be in relatively good condition as well as heavily damaged areas. Counter intuitively; the measurements adjacent to the fog line near the right wheel path indicated sound concrete (see the left b-scan in figure 7). The subsequent measurements indicated distress. An uneven backwall reflection at a shallower depth can be observed from scans at the center of the lane see the right b-scan in figure 7) indicating deterioration of the concrete at the interface with the base.

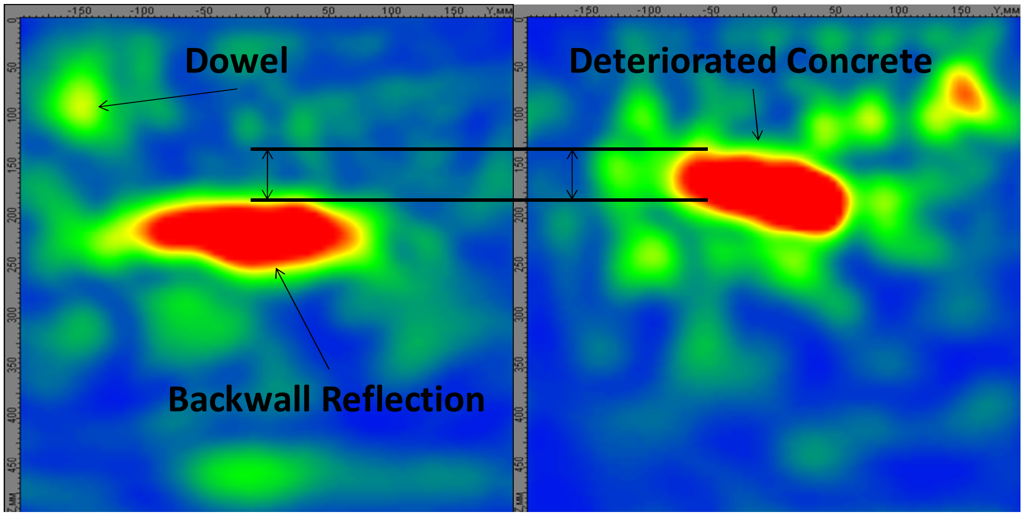

Figure 7. MIRA B-Scan locations in suspected sound (left b-scan) and deteriorating (right b-scan) conditions.

To further investigate the suspected damage, detailed testing was performed on a JPCP transverse joint before trenching. These scans showed, in greater detail, the trend of sound concrete in the right wheel path turning into deterioration as the scans were made closer to the centerline. A trench was made to verify the MIRA b-scan diagnosis. Figure 8 shows the interface where the trench was taken (top) and the flipped trench sections (bottom). It can be observed that the MIRA diagnostics were confirmed by the relatively good condition of the concrete on the underside near the right wheel path and subsequent deterioration of the concrete near the base in the trenches at the middle of the joint and at the centerline.

Figure 8. Trench locations (top) and flipped trench sections (bottom) showing concrete condition on the underside at various locations near the transverse joint.

There were also some MIRA b-scans at certain locations with a more uniform, shallow reflection (see the left b-scan in figure 9). These measurements indicated horizontal delamination and were marked for core verification at those locations. Forensic analysis of the cores and concrete condition at the interface created by the cores verified the delamination in these locations. Figure 9 illustrates an example b-scan location that indicated horizontal delamination. The b-scan on the right was taken adjacent to the suspected delaminating concrete with a reflection indicating sound concrete and a pavement interface at approximately 9 in. (~225 mm). The b-scan on the left, taken directly above the suspected delamination, shows a high intensity of reflection along the aperture near the surface (< 5 in.), with low reflection near the adjacent measured pavement depth (~9 in or 225 mm) most likely due to shadowing from the delamination. For this example location, figure 10 shows the core, interface, and some zoomed in shots of the delamination at the interface. It can be observed that the MIRA diagnostics were confirmed by horizontal crack at the core and interface at the depth indicated by the b-scan.

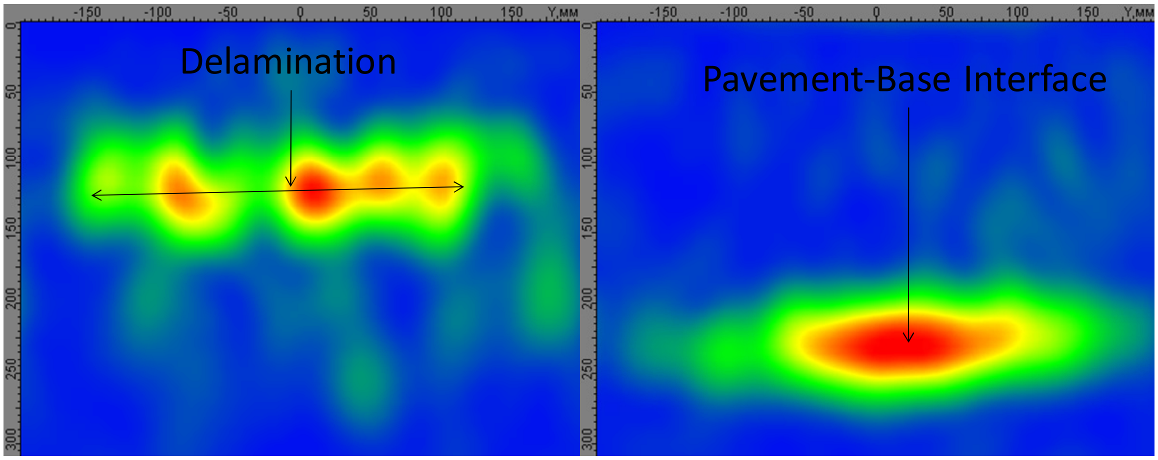

Figure 9. B-scans directly over delaminating concrete (left) and adjacent to the delamination with sound concrete (right).


Figure 10. Core showing delamination at the interior of the concrete.

AUTOMATION OF SIGNAL INTERPRETATION

While the above examples showed the potential of ultrasonic tomography to give accurate reinforcement concrete cover, the manual data interpretation procedure for this application was time consuming and labor intensive. In addition the interpretation process is somewhat subjective and requires a certain experience/expertise. To allow for more efficient objective analysis, software to automate analysis of typical applications should be developed. The software should be capable of identifying the object of interest (reinforcement location, pavement thickness interface, delamination, etc.). This can be done with the following general algorithm:

- Read the b-scan into the software
- Threshold high intensity of reflection areas from low intensity of reflection areas
- Identify characteristics of the type of reflection caused by the specific object of interest
- Use shape recognition schemes to decide if the identified areas are in fact the object of interest based on the identified characteristics
- Determine the location (depth and lateral position) of the center of the object of interest
- Eliminate false positives through a check with one or multiple adjacent scans
- Output the results to a spreadsheet along with information about the scan locations

This general outline was used to develop software capable of finding concrete cover over longitudinal rebar in CRCP. Figure 11 shows the steps (left top to bottom, then right top to bottom) in identifying 2 longitudinal rebars in an example b-scan. The figure illustrates the use of circularity criteria to identify longitudinal rebar reflections. After a threshold value of 0.80 is applied the two central reflections are identified as reinforcement, while the left and right reflections were rejected because their characteristics could not be determined reliably. Thus, in this case, the concrete cover for the identified center two reflections would be output to the spreadsheet.

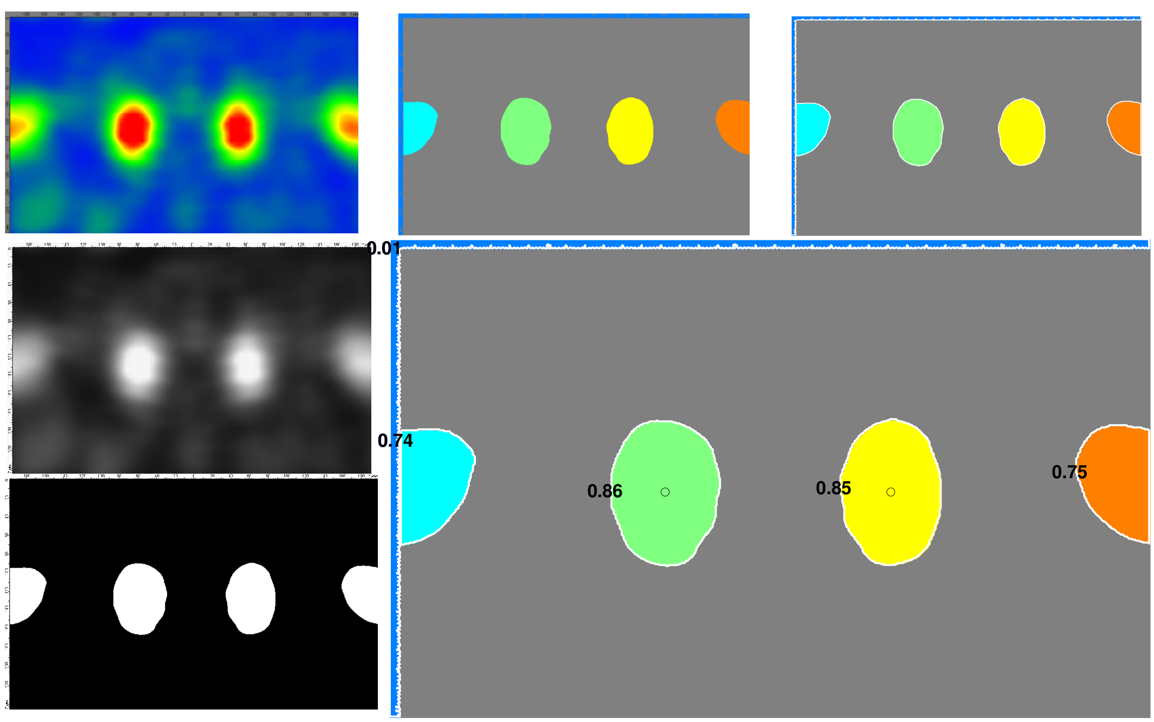

Figure 11. Progression in identifying the centroid of reflections caused by rebar.

Figure 12 shows MIRA measurements that were analyzed using the software versus core measured concrete cover at the same seven locations discussed earlier in the CRCP field trial. It can be observed that using an objective procedure to identify the centroid of longitudinal rebar reflections and calculate concrete cover created an even higher correlation ($R^2 = 0.991$ to $R^2 = 0.997$) and accuracy (slope = 0.972 to slope = 0.985) in the MIRA measurements. In addition, the automated procedure improved the analysis time. For example the same amount of CRCP data points that took about 2 weeks to manually analyze take less than 3 hours with the automated software.

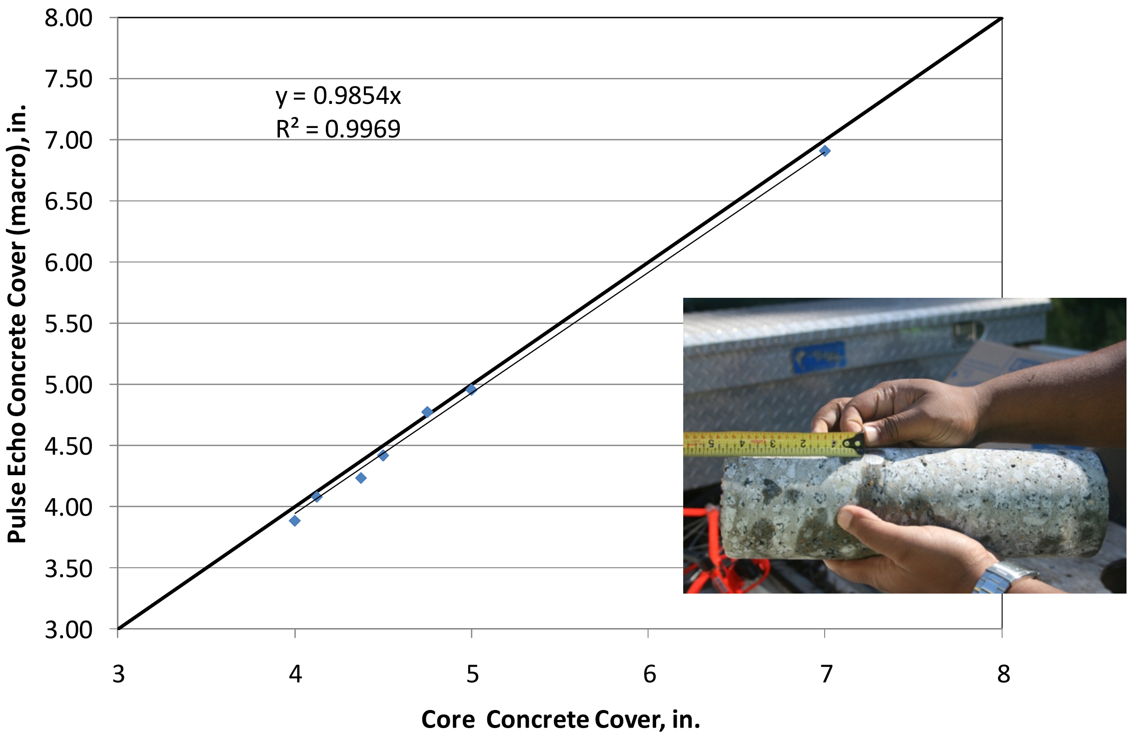

Figure 12.MIRA measured and software analyzed concrete cover versus core measured.

COMPARISON WITH OTHER NDT METHODS

As shown in this study, ultrasonic tomography has the ability to give real time and later high resolution analysis for multi-purpose applications. This device is a very portable option with a measurement process that is easily accessible to making pavement scans. The ultrasonic tomography method presented above offers a dramatic improvement compared to the traditional active acoustic methods such as impact echo. However, it is important to also compare this technique with NDT methods that are based on different physical principles. In recent years, ground penetrating radar (GPR) and magnetic based techniques have become widely used concrete pavement diagnostic techniques (Rao et al., 2009; Hoegh et al., 2008, Li et al., 2008).

Ground penetrating radar is capable of making ground or air coupled measurements to allow for high speed measurements with high penetration depths. The measurement and signal processing techniques have become very advanced in the ability to collect and analyze large pavement sections in a relatively short period of time. This allows for measurements at highway speed where road closure is not required. Although ultrasound tomography is not as effective as GPR on a network level, it can offer a higher resolution with respect to lateral position and depth location in certain cases. For example, if environmental conditions are such that there is a moisture gradient in the concrete, the dielectric constant and thus radar measurements could prove to be inconsistent while ultrasonic measurements would not. In cases such as this, MIRA can be used for GPR measurement verification or refining of GPR measurements on a project level.

One example of this type of procedure was conducted at a concrete pavement near Red Wing, MN. In this pavement network level ground coupled GPR measurements identified areas where tie bars were suspected to be missing at the longitudinal joints. MIRA was then used to verify the GPR diagnosis in an approximately 70 ft area (see figure 13). MIRA scans were also taken at the locations leading up to and after the 70 ft area identified by GPR measurements. MIRA scanning showed the presence and depth of tie bar reflections at the corresponding locations. Figure 13 shows a MIRA b-scan location with a tie bar reflection and pavement-base interface reflections. Using the MIRA b-scans, the depth and location of the reinforcements were found (see figure 14). It can be observed that the MIRA measurements generally agree with the GPR assessment of missing tie bars, with the exception of a few locations were tie bars were located by MIRA and not GPR. While GPR measurements did not identify the tie bars in these locations, it should be noted that GPR analysis has been proven to be successful in similar applications such as identification of tie bars when the correct radar type and interpretation method is used. Nonetheless, it can be concluded that there was a constructability issue causing the missing tie bars in locations where they were designed to be present.

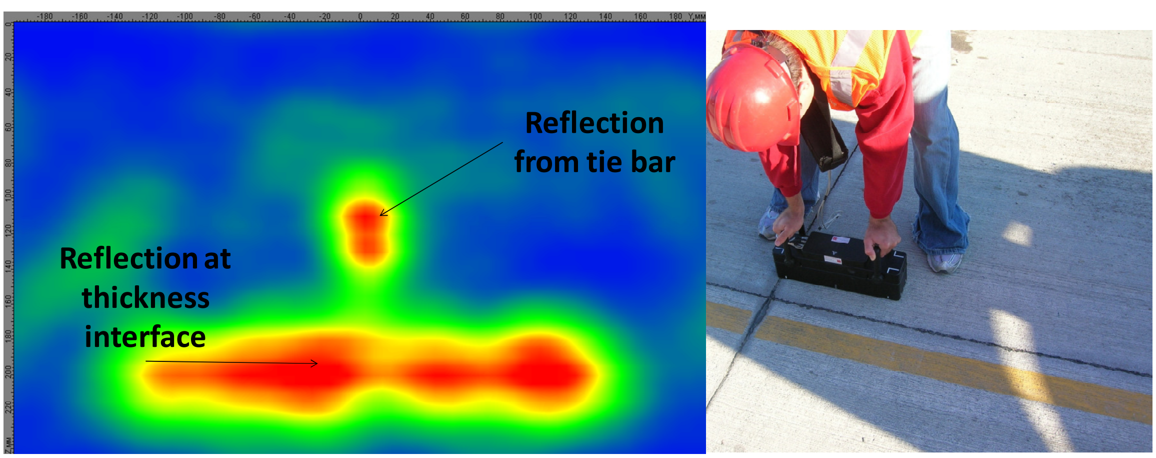

Figure 13. MIRA measurements along longitudinal joints to identify tie bars.

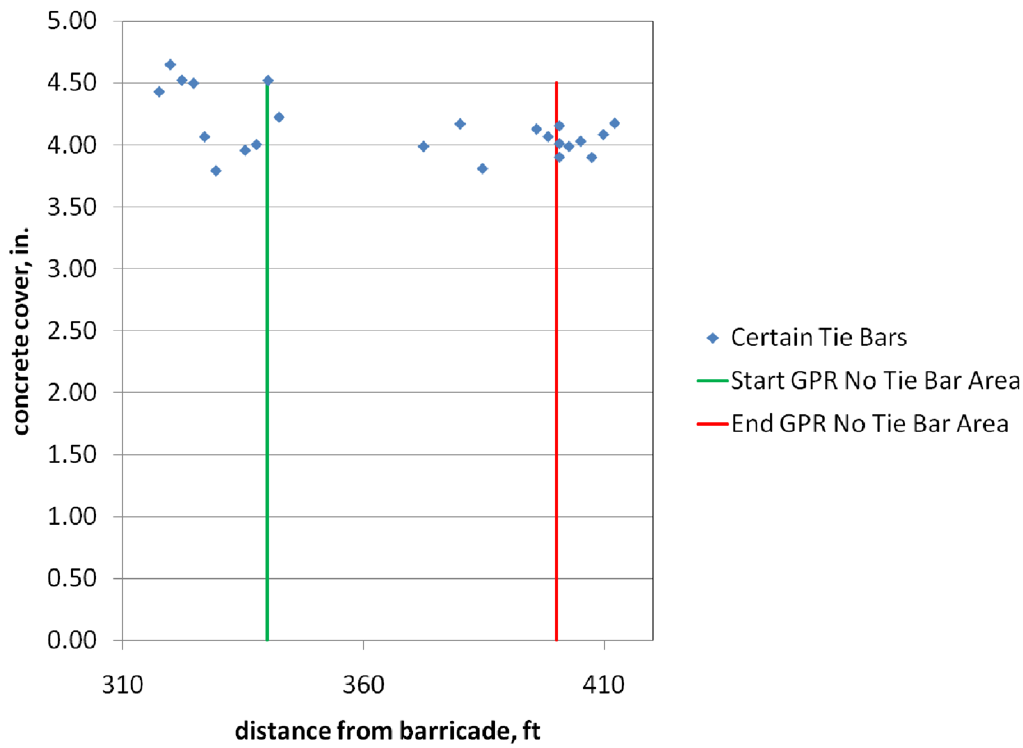

Figure 14. MIRA measurements of tie bar presence and concrete cover versus location along an area diagnosed by GPR to have missing tie bars.

Extremely precise location of metal reinforcements can be achieved using magnetic techniques. In addition, one magnetic device, MIT-SCAN 2, has proven to be proficient in large scale measurements of dowel location in transverse joints (Rao et al., 2009; Hoegh et al., 2008). Although MIRA can be used for detection of dowel misalignment, its productivity is not comparable with MIT-SCAN 2. Nevertheless, it can be used for initial screening to identify a need for more comprehensive MIT-SCAN 2 evaluation. It is also important to point out that while MIT-SCAN 2 can only be used for dowels, preferably installed with a dowel bar inserter, MIRA can be used for various other applications as well.

As described above, each of the NDT techniques have their own strengths and weaknesses. None of the techniques are able to completely address all the concrete pavement diagnostic needs. The value of each individual technique can be significantly improved through applying several individual techniques together with subsequent data fusion of the results from each method.

For example, GPR depth estimates need to be calibrated based on the dielectric constant of the tested material. Traditionally this is done using cores. Ultrasound tomography could be used to eliminate the need for coring and increase the frequency of calibration points. In addition, GPR may not be able to distinguish between layers with similar dielectric properties, but if the layers have different acoustic properties, ultrasound tomography can be used in conjunction with GPR to resolve this issue.

MIT-SCAN 2 is capable of locating dowels with high precision regardless of concrete layer conditions because the technique is unaffected by non-metal mediums. Because of this, MIT-SCAN 2 is not capable of assessing the condition of concrete around the dowel. MIRA is capable of identifying dowel locations and evaluation of damage in the surrounding concrete. However, with knowledge of the precise location of the dowels, a higher resolution analysis of the concrete condition around dowels could be made with MIRA.

The examples above show that data fusion of the results of nondestructive testing methods utilizing different principles is a very promising direction of pavement diagnostics. Future research in this area has great potential.

CONCLUSIONS

A recently developed ultrasonic tomography device (MIRA) has shown to be an attractive NDT tool for concrete pavements. In this study the device was demonstrated to be capable of determining concrete pavement thickness and reinforcement location. It was also shown to be proficient in detection of flaws such as delamination and degradation at pavement joints. The device comes with powerful data interpretation software that utilizes the synthetic aperture focusing technique. However, to ensure more widespread implementation, it is important to customize and automate the data interpretation process for the most typical applications. This automation will reduce the need for specialized user expertise/experience and improve the productivity and objectivity of data interpretation.

Although MIRA can be used as a stand-alone tool, it can also be effectively used in combination with other NDT methods. This type of data fusion will significantly improve the quality of pavement NDT evaluation.

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